

Optimal taxation and borrowing constraints*

Estructura impositiva óptima y restricciones de crédito

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Abstract

I propose a new overlapping generations model, in which individuals face different income levels, life expectancies and borrowing constraints to study Ramsey optimal taxation. Contrary to previous contributions, I find that optimal capital income taxation generally differs from zero in the long term even when preferences are additively separable. I also find that the tax system should generally incorporate a progressive capital income tax in the long run. Furthermore, the model enables to disentangle the respective roles of finite life horizons, productivity differences and borrowing limits.

Key words: Optimal taxation, borrowing constraints, overlapping generations.

Resumen

Se propone un nuevo modelo de generaciones solapadas para estudiar la fiscalidad óptima de Ramsey en el que los individuos enfrentan diferentes niveles de ingresos, esperanza de vida y restricciones de crédito. En contraste con la literatura previa, el artículo sugiere que el impuesto óptimo a los retornos del capital generalmente difiere de cero en el largo plazo aun para el caso de preferencias aditivamente separables. El artículo también encuentra que el sistema fiscal en general debe incorporar un impuesto progresivo sobre la renta de capital en el largo plazo. Además, el modelo permite separar y evaluar las funciones respectivas de los horizontes de vida finita, las diferencias de productividad y los límites de endeudamiento.

Palabras clave: imposición óptima, restricciones de crédito, mercados incompletos.

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1. Introduction

How should a stream of exogenous government expenditure be financed using capital and labor income taxation? This is a standard question in the Ramsey optimal taxation literature. In their seminal papers, Chamley (1985, 1986) and Judd (1985, 1987) find that capital income taxation should equal zero in the long run and should not be used for redistribution. My contribution essentially builds upon Erosa and Gervais (2002) who have challenged these results. In a simple life-cycle model with endogenous labor supply, Erosa and Gervais (2002) show that, as consumption increases with age, it is generally optimal to use age-dependent capital income taxation in the steady state. More precisely, they show that capital income taxation differs from zero when the elasticity of the marginal utility of consumption is not constant in the steady state. With a flat labor productivity age-profile, the limitation to this result is that capital income taxation should remain equal to zero in the widely used case of additively separable preferences. I propose to study Ramsey optimal taxation in a new overlapping generations model with endogenous labor supply. I determine the theoretical conditions for non-zero capital income taxation in the steady state and its redistribution implications. In the proposed framework, individuals face different income levels, life expectancies and borrowing constraints. I find that the conditions for zero optimal capital income taxation are more restrictive than in the simple life-cycle model. Optimal capital income taxation generally differs from zero in the long term even when preferences are additively separable. I also find that the tax system should generally incorporate a progressive¹ capital income tax in the long run. Furthermore, the model enables to disentangle the respective roles of finite life horizons, income differences and borrowing limits.

This paper is a theoretical contribution which does not target a specific economy. However, I think that this paper is of particular interest for Latin America. Income and wealth inequalities remain high in Latin America (Lopez-Calva and Lustig, 2010; Klasen and Nowak-Lehmann, 2009). Similarly, borrowing limits are significant in many Latin American countries (Inter-American development bank, 1998-

1999). Brazil, the largest economy in Latin America, is a striking example. The 2010 UN Human Development Report shows that for the past twenty years, Brazil has remained one of the top five nations with the largest income inequalities in Latin America. The existence of borrowing constraints is also well-documented for Brazil (*e.g.* Duryea, 1998). Additionally, the study by Bernardi *et al.* (2007)² shows that over the past twenty years, many Latin American countries have decreased their reliance on income taxes and trade taxes relative to consumption taxes. For example, Brazil has low tax revenues by international standards from income and profits taxation (7.9% of GDP), and assets taxation (1.2% of GDP). It has high tax revenues from consumption taxation (19.5% of GDP). Consumption and salaries taxation alone represent more than 70% of tax revenues in Brazil.³ Thus, recent reforms in Latin America seem consistent, to some extent, with past theoretical arguments of closed and open economy models in favor of eliminating capital income taxation.⁴ However, capital income taxation remains positive in Latin American countries. Therefore, the proposed model provides a tool to re-think of optimal taxation in an environment that captures some of the important characteristics of Latin American countries.

The proposed theoretical model is an extension of a workhorse of macroeconomics, the Yaari (1965)-Blanchard (1985) overlapping generations model. This benchmark model is built on the following premises. All individuals face the same life expectancy and can insure their constant mortality risk with life annuities. Credit markets are complete and all individuals can therefore smooth consumption over their lifetime. However, empirical studies show that individuals differ with respect to annuitization. Dushi and Webb (2004) find that contrary to the rich, the poor do not annuitize. In the first place, actuarial unfairness induces a delay in annuitization and a reduction in the amounts annuitized. In the second place, if wealthier households believe that they have a longer life expectancy than average, they increase the value they place on annuitization. The empirical literature also shows that rich individuals have a longer life expectancy than poor individuals. This point has been documented by Waldron (2007) who relates life expectancy with average earnings and by Attanasio and Hoynes (2000)

who relate wealth and longevity. Finally, credit markets might not be complete. For Aghion *et al.* (1999) the existence of borrowing limits by lenders comes from the simple fact that some borrowers may choose not to repay their loans. In addition, recent empirical results by Crook and Hochguertel (2007) show that wealth reduces the chance of being credit constrained.

Taking into consideration those findings, I modify the Yaari (1965)-Blanchard (1985) framework as follows. I introduce two categories of agents, the rich and the poor. The rich are born with no assets on a high income path and have a long life expectancy. The poor are born with no assets, on a low income path and have a shorter life expectancy. The model presupposes that the rich individual has access to a life annuity market as in the Yaari (1965)-Blanchard (1985) setting: In case the rich individual dies holding wealth, her wealth goes to a non-profit mutual fund. She gets a payout during her lifetime in exchange for returning her wealth to the mutual fund contingent on her death. As a consequence, contrary to the infinite horizon model, in the steady state, the return on her wealth is superior to her discount rate.⁵ Thus, the rich individual has an increasing consumption profile and accumulates wealth. I presuppose that individuals face borrowing limits. As it is not optimal for the rich individual to borrow, borrowing constraints are non-binding. By contrast, the low income individual does not annuitize. As a consequence, the return on her wealth is inferior to her discount rate. The poor individual wishes to borrow and her borrowing constraint binds.

In the model, in the absence of income uncertainty, precautionary saving is ruled out⁶ and there is only life-cycle saving by the rich. In a similar way to Blanchard (1985), the labor income age profile is either flat or decreasing to reflect retirement. While uncertainty about death increases aggregate consumption, the steepness of the decrease in income profile stimulates aggregate saving. Supposing that the latter effect does not dominate, the aggregate economy is dynamically efficient. If the economy is dynamically efficient, the role of capital income taxation is not to limit capital accumulation as in Ayagari's (1995) model with precautionary saving. In the absence of a precautionary saving

motive, individuals faced with different borrowing limits have different consumption and saving behaviors. The role of age-dependent capital income taxation is therefore to reallocate consumption not only over the life-cycle of the poor, but also across the poor and the rich categories contrary to Conesa *et al.* (2007, 2009).⁷

In this framework, I determine the “second best” (Ramsey) optimal taxation scheme, assuming commitment by the government to rule out time inconsistency: Age-dependent labor and capital income taxes are raised to finance an exogenous public spending sequence. Once optimal tax rates are determined, the government adjusts the level of debt to satisfy the period budget constraint. The government chooses the set of labor and capital income taxes that maximizes a social welfare function taking into consideration the rich and the poor’s well-being. Because the government cares about the poor, the tax combination that aims at maximizing efficiency also involves redistribution. I solve the model combining the primal approach inspired by Atkinson and Stiglitz (1972, 1980) and the two-step method by Calvo and Obstfeld (1988) for the perpetual youth model. I write the analytical expressions for optimal labor and capital income taxes to determine in which environment capital income taxation should be used.

The proposed model presents the advantage of being flexible to consider a wide range of scenarios and disentangle the roles of income differences, borrowing constraints and different life horizons. The results, presented in *Propositions 1 to 6* and their corollaries are summarized as follows. First, the model enables to study a finite horizon version of the “workers-capitalist” model by Judd (1985). Contrary to Judd (1985), I find that in a finite horizon framework, capital income taxation faced by “capitalists” generally differs from zero even when preferences are additively separable. The optimal capital income tax faced by “workers” (consistent with leaving their behavior unchanged) is zero. The optimal capital income tax scheme is generally progressive across income categories. In a second scenario, the optimal tax scheme involves a behavioral response by the poor and results in loosened borrowing constraints. Capital income taxation faced by the rich generally differs from zero. However, compared to the previous scenario, there are

additional cases in which capital income taxation faced by the rich is zero. The optimal tax scheme is generally progressive and the poor's return on wealth can be taxed or subsidized. In a third scenario, I relax some assumptions of the model to consider non-binding borrowing constraints in the *initial* steady state. Assuming that both the poor and the rich annuitize, borrowing constraints do not bind. The poor and the rich differ only by their income profiles. Compared to the second scenario, the conditions for non-zero optimal capital income taxation for the rich are similar. When it is optimal to subsidize the poor individual's capital income, the optimal subsidy to the poor is smaller than in the absence of annuitization (second scenario). By simply modifying the life expectancy parameters, the proposed framework also enables me to study various scenarios regarding the life horizon. The model embeds the results by Erosa and Gervais (2002) and Judd (1985). In addition, I find that the conditions for non-zero capital income taxation in the steady state are generally reinforced when different life horizons are taken into consideration. However, using logarithmic preferences, I show that when the life horizon of the rich becomes infinitely large relatively to the poor, the capital income tax paid by the rich converges toward zero but it generally remains optimal to subsidize the poor's capital income.

The paper is organized as follows. In the second section, I review the literature. In the third section, the formal framework is presented. In the fourth section, I present the theoretical results.

2. Literature review

The results of the Ramsey benchmark literature pertain to the choice of an infinite horizon.⁸ In those models, to finance a given amount of government expenditure, only distortionary tax instruments are available to the government. As a consequence, the government has an incentive to levy a one-time tax on the existing (inelastic) stock of capital of the economy and to not tax capital thereafter. By doing that, the government prevents distortions and the economy can reach a "first-best" Pareto optimum rather than a "second-best".

The literature about optimal taxation has developed in various directions to challenge the result of zero capital income taxation. This review exclusively focuses on two types of contributions in the Ramsey optimal taxation literature, simple overlapping generations (OLG) models or models incorporating incomplete markets.⁹

In the dynamic framework of OLG models, we can distinguish between two types of results. In one (see *e.g.* Ambler, 1999), derived from the Diamond (1965) model, the market economy can accumulate too much capital compared to the golden rule. This yields a steady state equilibrium that is not Pareto efficient. Taxing capital income can therefore improve efficiency in the first place. In the other (see *e.g.* Erosa and Gervais, 2002; Garriga, 2003, or Mathieu-Bolh, 2006), the economy accumulates capital and reaches a second-best Pareto efficient equilibrium. If the government takes into consideration the well-being of current and future generations, it has a motive to postpone capital taxation to the future to some extent. As a result, in the steady state, using age-dependent taxes, it is optimal to tax individuals differently according to the elasticity of their marginal utility of consumption, which entails redistribution across age cohorts. With a constant labor productivity age-profile, the limitation to this result is that capital income taxation should remain equal to zero in the case of additively separable preferences (Atkeson *et al.*, 1999).

When unemployment insurance markets are incomplete, Aiyagari (1995) and Chamley (2001) find that it is optimal to tax capital income in the long run. In both models, agents are optimizing over an infinite horizon and face random income shocks, against which they self-insure. As a consequence, if the market economy accumulates too much capital compared to the golden rule due to precautionary saving, the optimal capital income tax is positive in the long run.

Conesa *et al.* (2007, 2009) quantitatively characterize capital income taxation in a life-cycle model with uninsurable income shocks and permanent productivity differences of households. First, they find that capital income taxation is generally positive and the high capital income tax relates to the life-cycle structure of their model. In a life-cycle model with endogenous labor supply, it is optimal to tax labor at

different ages at different rates. If the government does not have access to an age-dependent labor income tax, a positive capital income tax can achieve the same as a progressive labor income tax. However, when labor is endogenously supplied and the labor income tax system depends on age, they find that capital income taxation equals zero when preferences are additively separable between consumption and leisure, and that redistribution is achieved by labor income taxation. Second, they find that market incompleteness and distributional concerns determine the optimal progressivity of the labor income tax.

My paper proposes an overlapping generations framework with incomplete credit markets. It extends the study of Ramsey optimal taxation in life-cycle models (*i.e.* Erosa and Gervais, 2002) by taking into consideration individuals with different income profiles, life expectancies and borrowing constraints. At first, the introduction of incomplete credit markets in infinite horizon models seemed to prove the robustness of the traditional results. Judd (1985) and Mankiw (2000) show that capital income taxation is undesirable in the long run because it decreases the real wage. Workers prefer the static distortion of marginal labor income taxes to the cumulative distortion of the capital income taxes on intertemporal margins. My results contrast with Judd (1985) and Mankiw (2000) by showing that in a finite horizon model incomplete credit markets modify the traditional recommendation of not taxing capital income in the long term. Moreover, the conditions for optimal capital income taxation to be zero in the long run are tighter than in Erosa and Gervais (2002). I show that optimal capital income taxation generally differs from zero in the long run even when preferences are additively separable. In addition, I find that the optimal capital income tax scheme is generally progressive. Therefore, the results also contrast with Conesa *et al.* (2007, 2009). This contrast comes from the difference in the consumption and saving behavior of individuals. In Conesa *et al.* (2007, 2009), all agents constitute precautionary saving in anticipation of adverse income shocks. In my model, agents have different consumption and saving behaviors in the presence borrowing limits. A progressive capital income tax scheme can be used to loosen borrowing constraints, enabling poor agents to smooth consumption

over the life-cycle and the social planner to reallocate consumption across income categories. The effect of the borrowing constraints on optimal labor income taxation is ambiguous.

3. The formal framework

3.1. The economy

3.1.1. Population structure

In this model, time is continuous. There are two types of individuals, poor and rich. To normalize the size of the total population to one, I assume that the cohort size at birth of poor individuals equals $\frac{1}{\lambda_p + \lambda_R}$. The poor face a constant rate of death $\frac{1}{\lambda_p}$. Time until death has an exponential distribution. Therefore, on a date t the size of a cohort of poor individuals born in year s is $\frac{1}{\lambda_p + \lambda_R} e^{-\frac{1}{\lambda_p}(t-s)}$. Integrating on s , the date of birth since $-\infty$, the size of the population of the poor on a date t is:

$$\int_{-\infty}^t \frac{1}{\lambda_p + \lambda_R} e^{-\frac{1}{\lambda_p}(t-s)} ds = \frac{\lambda_p}{\lambda_p + \lambda_R}$$

Each instant, a cohort of size $\frac{1}{\lambda_p + \lambda_R}$ of rich individuals is born. The rich face a constant rate of death $\frac{1}{\lambda_R}$. Therefore, on a date t , the size of a cohort of rich individuals is $\frac{1}{\lambda_p + \lambda_R} e^{-\frac{1}{\lambda_R}(t-s)}$. As a result, the size of the population of the rich is:

$$\int_{-\infty}^t \frac{1}{\lambda_p + \lambda_R} e^{-\frac{1}{\lambda_R}(t-s)} ds = \frac{\lambda_R}{\lambda_p + \lambda_R}$$

The size of the entire population is therefore equal to:

$$\frac{\lambda_p}{\lambda_p + \lambda_R} + \frac{\lambda_R}{\lambda_p + \lambda_R} = 1$$

By assumption, each cohort is an infinitely divisible continuum of agents, such that each cohort of poor agents decreases at a constant rate $\frac{1}{\lambda_p}$ and each cohort of rich agents decreases at a constant rate $\frac{1}{\lambda_R}$. There is, therefore, no aggregate uncertainty and the size of the population of the poor and the rich is constant.

3.1.2. The behavior of consumers

The individual behavior of the poor and the rich can be described by the following optimization program. The utility of an individual of type i ($i = P$ for poor or R for rich) equals $U[C^i(s, z), 1 - N^i(s, z)]$. The variable $C^i(s, z)$ denotes consumption on date z of an individual born on date s . Individuals are endowed with one unit of time, that they share between work $N^i(s, z)$ and leisure, $L^i(s, z)$. The objective of an individual is:¹⁰

$$\max_{C^i(s, z), N^i(s, z), W(s, z)} \int_t^{+\infty} e^{-(\delta + \frac{1}{\lambda_i})(z-t)} U[C^i(s, z), 1 - N^i(s, z)] dz$$

st.:

$$\dot{W}^i(s, z) = \left[\theta_i \left(\frac{1}{\lambda_i} \right) + \hat{r}(z) \right] W^i(s, z) + \hat{\omega}^i(z) E^i(z - s) N^i(s, z) - C^i(s, z)$$

and a borrowing constraint:¹¹

$$W^i(s, z) \geq -D^i(s, z)$$

The rate of time preference is δ . Financial wealth $W^i(s, z)$ is composed of government bonds $B^i(s, z)$ and shares of capital $K^i(s, z)$. Shares and bonds are perfect substitutes and wealth provides an after-tax return equal to $\hat{r}(z) + \theta_i \left(\frac{1}{\lambda_i} \right)$, where $\hat{r}(z)$ is the after tax rate ($\hat{r}(z) = r(z)(1 - T_k(z))$). Since the length of life is uncertain, individuals can die leaving unintentional bequests. Following Yaari (1965) and Blanchard (1985), I introduce a perfect annuity market. The parameter θ_i denotes the access to the annuity market. If individuals of type i have access to the annuity market, $\theta_i = 1$. For example, with $\theta_R = 1$, each instant a non-profit mutual fund collects the financial wealth W^R from rich deceased individual. Since a proportion $\frac{1}{\lambda_R}$ of rich individuals die, the mutual fund collects $\frac{1}{\lambda_R} W^R$ from the deceased. At the same time, the mutual fund pays $\frac{1}{\lambda_R} W^R$ to the rich currently alive. The rich get this payout in exchange for returning their wealth to the mutual fund contingent on their death.¹² Therefore, the term $\theta_i \left(\frac{1}{\lambda_i} \right) W^i(s, z)$ represents the insurance payout. If individuals do not annuitize, $\theta_i = 0$ and there is no insurance payout. Individuals pay a labor-income tax $T_\omega^i(z)$, which is a fraction of their real wage $\omega(z)$. The after tax real wage $\omega(z)(1 - T_\omega^i(z))$ is simply

denoted $\hat{\omega}^i(z)$. It represents the remuneration of one unit of effective labor. The labor income age-profile is given by a function $\tilde{N}^i(z-s)$ (see section 3.1.3.). $E^i(z-s)$ denotes labor efficiency. The borrowing limit is given by $D^i(s,z)$. Debt $(-W^i(s,z))$ cannot exceed $D^i(s,z)$.

The program is solved in a standard way. Given $\mu_1^i(s,z)$ the multiplier associated with the budget constraint, and μ_2^i the multiplier associated with the borrowing constraint, the optimality conditions are:

$$\begin{aligned}\mu_1^i(s,z) &= U_{C^i(s,z)} \\ E^i(z-s)\hat{\omega}^i(z) &= -\frac{U_{N^i(s,z)}}{U_{C^i(s,z)}} \\ \dot{\mu}_1^i(s,z) &\leq \left(\delta + (1-\theta_i)\frac{1}{\lambda_i} - \hat{r}(z) \right) \mu_1^i(s,z) \\ \lim_{z \rightarrow +\infty} \mu_1^i(s,z)W^i(s,z) &= 0 \\ \mu_2^i(W^i(s,z) + D^i(s,z)) &= 0\end{aligned}$$

I first consider the base case when only the rich annuitize, $\theta_R = 1$ and $\theta_p = 0$. At the individual level, because of the annuity market for life insurance, the consumption-saving intertemporal choice of the rich depends on $\delta - \hat{r}$. It does not depend on $\frac{1}{\lambda_R}$ because the rich accumulate wealth at a rate $\frac{1}{\lambda_R} + \hat{r}$ and their discount rate depends on $\delta + \frac{1}{\lambda_R}$. By contrast, the poor do not annuitize and their intertemporal choice depends on $\delta + \frac{1}{\lambda_p} - \hat{r}$.

If $\delta < \hat{r}$, the rich individual has an increasing consumption age-profile and therefore accumulates wealth. As a consequence, $W^R(s,z) > 0$ and $\mu_2^R = 0$. For the rich individual, the borrowing constraint is not binding and the Euler equation holds as an equality. This assumption is consistent with the study of the dynamics and steady state (see section 3.2). In a similar way to Blanchard (1985), the study of the dynamics and steady state shows that for the economy to reach the steady state, some individuals must save. A simple scenario is that the rich are the ones who save. If the rich save, their individual consumption must grow

on the saddle path and in the steady state, where $\delta < \hat{r} \leq \delta + \frac{1}{\lambda_r} \leq \delta + \frac{1}{\lambda_p}$. Then, at the same time, with $\hat{r} \leq \delta + \frac{1}{\lambda_p}$ the poor individual does not accumulate wealth. The poor individual desires to consume at least her income. I consider the case when $D^i(s, z) = 0$ which is a no-borrowing constraint. In the presence of a no-borrowing constraint the poor individual consumes exactly her disposable income. As a consequence, $W^P(s, z) = 0$ and $\mu_2^P \neq 0$.

The observation of the steady state condition $\delta < \hat{r} \leq \delta + \frac{1}{\lambda_r} \leq \delta + \frac{1}{\lambda_p}$ provides a first insight on the optimal taxation scheme. Welfare gains from consumption smoothing can be achieved if the poor are relieved from the binding borrowing constraint. If the rich pay a capital income tax ($T_k^R > 0$) there is less capital accumulation and a higher interest rate in equilibrium. If the effect on the interest rate is large enough, the interest rate faced by the poor becomes larger than their discount rate. The poor's optimal choice is to save, which relieves them from the binding borrowing constraint. However, if the poor save, this has a positive effect on capital accumulation and decreases the before-tax interest rate, which partially or totally cancels out the effect of the capital income tax on the rich. The capital income tax on the rich needs to be associated to a capital income subsidy to the poor ($T_k^P < 0$) in order to keep the after tax interest rate \hat{r}^P larger than the poor's discount rate.¹³

Second, I consider the case when both the poor and the rich annuitize, $\theta_i = 1$. As a result, the Euler condition for the rich and the poor is written in the same way:

$$\dot{\mu}_1^i(s, z) = (\delta - \hat{r}(z))\mu_1^i(s, z)$$

With $\hat{r} > \delta$, poor and rich individuals save. As a result, borrowing constraints never bind. Then, along an optimal path $\mu_2^i \neq 0$ and the Euler equation holds as an equality. This scenario enables to isolate the role of different income levels from the role of borrowing constraints.

When individuals can smooth consumption, the intertemporal budget constraint and the Euler equation can be used to express individual consumption as a function of human and financial wealth. Thus, the following equation describes the behavior of the rich in the

base case and of both the poor and the rich in the case of non-binding borrowing constraints:

$$C^i(b, t) = \varphi^i(b, t)[H^i(b, t) + W^i(b, t)] \quad (1)$$

where $\varphi^i(b, t)$ represents the average propensity to consume, $W^i(b, t)$ represents financial wealth and $H^i(b, t)$ human wealth. Human wealth is the present discounted value of future after-tax labor income.

$$H^i(s, z) = \int_t^{+\infty} e^{-\int_t^v [\rho^i(v) + \frac{1}{\lambda_i}] dv} \hat{\omega}^i(z) E^i(s, z) N^i(s, z) dz$$

3.1.3. The aggregate economy

Aggregate variables are deduced from individual behavior and from the aggregation rules derived from the population's structure. For the poor, the link between an individual variable $X^P(s, t)$ and an aggregate variable $X^P(t)$ is:

$$X^P(t) = \int_{-\infty}^t \frac{1}{\lambda_p + \lambda_R} e^{-\frac{1}{\lambda_p}(t-s)} X^P(s, t) ds$$

For the rich, the link between an individual variable $X^R(s, t)$ and an aggregate variable $X^R(t)$ is:

$$X^R(t) = \int_{-\infty}^t \frac{1}{\lambda_p + \lambda_R} e^{-\frac{1}{\lambda_R}(t-s)} X^R(s, t) ds$$

For the entire population, aggregate variable X is:

$$X(t) = X^P(t) + X^R(t)$$

In order to aggregate the model and provide an analytical solution, I need to make the distribution of labor income across ages explicit, with "relative labor income functions" assuming exponential functional forms.¹⁴ The age-distribution of labor income $\hat{\omega}^i(t)E^i(s, t)N^i(s, t)$ is identical to the age-distribution of efficient labor $E^i(s, t)N^i(s, t)$. The variable $\tilde{N}^i(s, t)$ denotes efficient labor on a date t for an individual born in s :

$$\tilde{N}^i(s, t) = E^i(s, t)N^i(s, t)$$

The distribution of the efficient labor is given by:¹⁵

$$\tilde{N}^i(s, t) = a_i e^{-\alpha_i(t-s)} E^i(t)N^i(t)$$

where aggregate efficient labor is $E^i(t)N^i(t)$. The parameters of the exponential functional forms are chosen to reflect that the labor income of the poor is lower than labor income of the rich. Those functions are used to derive aggregate labor supply and the dynamic equation for human wealth (See online Appendix).

When the poor face binding borrowing constraints (base case), they consume their disposable income and their aggregate consumption is:

$$C^l(t) = \hat{\omega}(t)\tilde{N}^l(t)$$

When individuals face non-binding borrowing constraints, I use (1) to find the expression of their aggregate consumption. If we assume the average propensity to consume is independent of the date of birth,¹⁶ aggregate consumption can be made explicit and equals:

$$C^i(t) = \varphi^i(t)[H^i(t) + W^i(t)] \tag{2}$$

where $H^i(t)$ denotes aggregate human wealth and $W^i(t)$ denotes aggregate financial wealth and are equal to:

$$\dot{H}^i(t) = \left(\hat{r}(t) + \frac{1}{\lambda_i} + \alpha_i \right) H^i(t) - \hat{\omega}^i(t) E^i(t) N^i(t) \tag{3}$$

$$\dot{W}^i(t) = \hat{r}(t)W^i(t) + \hat{\omega}^i(t) E^i(t) N^i(t) - C^i(t) \tag{4}$$

Because of the annuity system, the accumulation of aggregate financial wealth does not depend on the occurrence of deaths (see Blanchard, 1985).

Firms produce $Y(t)$, using aggregate capital $K(t)$ and effective labor $E(t)N(t)$. The production function is Cobb-Douglas:

$$Y(t) = F[K(t), E(t)N(t)] = [K(t)]^\phi [E(t)N(t)]^{1-\phi}$$

At each instant, firms maximize their instantaneous profit. Assuming perfect competition, production factors (capital and effective labor) are paid their marginal product (respectively $\omega(z)$ and $r(z) + d$). Investment is described by:

$$I(t) = \dot{K}(t) + dK(t)$$

where d is the rate of capital depreciation ($d \in [0, 1]$).

The government finances exogenous public spending $G(t)$ by means of taxes $T(t)$ and debt $B(t)$. For any value of the tax rates, the debt level adjusts to satisfy the dynamic budget constraint:

$$\dot{B}(t) = \hat{r}(t)B(t) + G(t) - T(t)$$

with:

$$T(t) = T_\omega(t)\omega(t)E(t)N(t) + T_k(t)r(t)K(t)$$

The model is closed with the market clearing condition:

$$\dot{K}(t) = (F'_K(t) - d)K(t) + F'_{EN}(t)E(t)N(t) - C(t) - G(t)$$

3.2. Dynamics and steady state

To provide a straightforward analytical demonstration and some intuition for the main results of the paper, I use a CRRA with $\sigma_{c^h} = 1$ (logarithmic preferences). It is then straightforward to describe the dynamics of aggregate consumption by the rich. In the base case, the poor hold no wealth; the rich hold all the wealth in the economy, therefore $W^R(t) = W(t)$. Starting from the static expression:

$$C^R(t) = \left(\delta + \frac{1}{\lambda_R} \right) (W(t) + H^R(t))$$

I derive this expression and use equations (2), (3) and (4) to obtain the dynamics of aggregate consumption by the rich:

$$\dot{C}^R(t) = (\hat{r}(t) + \alpha_R - \delta)C^R(t) - \left(\delta + \frac{1}{\lambda_R} \right) \left(\frac{1}{\lambda_R} + \alpha_R \right) W(t)$$

To describe the model with two simple dynamic equations, without loss of generality, I can make some additional simplifications: There is no capital depreciation ($d = 0$), tax revenues come entirely from capital income taxation ($T = rT_k K$) and the budget is balanced. Therefore: $r = F'_k$, $G = rT_k K$ and $B = 0$. The dynamics of the model is therefore described by:

$$\frac{\dot{C}^R(t)}{C^R(t)} = [F'_K(t)(1 - T_K(t)) + \alpha_R - \delta] - \left(\delta + \frac{1}{\lambda_R} \right) \left(\frac{1}{\lambda_R} + \alpha_R \right) \frac{K(t)}{C^R(t)} \quad (5)$$

$$\dot{K}(t) = F'_K(t)(1 - T_k(t))K(t) + F'_{\tilde{N}}(t)(1 - T_\omega(t))\tilde{N}(t) - C^R(t) - C^P(t)$$

with:

$$C^P(t) = F'_{\tilde{N}}(t)(1 - T_\omega(t))\tilde{N}^P(t)$$

Contrary to the infinite horizon model, the dynamics of aggregate consumption by the rich differs from the dynamics of their individual consumption. Indeed, aggregate consumption growth for the rich depends on two terms. The first $(F'_K(t)(1 - T_K(t)) + \alpha_R - \delta)$ is the growth rate of their individual consumption. The second $-\left(\delta + \frac{1}{\lambda_R}\right)\left(\frac{1}{\lambda_R} + \alpha_R\right)\frac{K(t)}{C^R(t)}$ is an additional term which causes a drag on aggregate consumption growth. With uncertain life horizon, the rich die holding a positive wealth, whereas new generations are born with no financial wealth. Therefore, this additional term denotes the turnover of generations. The dynamics of aggregate consumption by the rich is therefore similar to the dynamics of total aggregate consumption in the Blanchard (1985) model. The equilibrium (besides the origin) is saddle point stable and the saddle path indicates that the aggregate economy accumulates wealth and that consumption is increasing. In the steady state, the stock of capital is such that:

$$\delta - \alpha_R < F'_K(t)(1 - T_K(t)) < \delta + \frac{1}{\lambda_h}$$

The upper bound for $F'_K(t)(1 - T_K(t))$ is a sufficient condition for the concavity of F . The lower bound is directly deduced from (5) (see online Appendix). The steady state condition implies that some

individuals must accumulate wealth. When $\alpha_R = 0$, a simple scenario consistent with these inequalities is that the rich have an increasing consumption profile and therefore accumulate wealth. If $\alpha_R > 0$, the rich still need to accumulate wealth early in life and therefore $\delta < \hat{r}(z)$. If at some point, $\delta - \alpha_R < \hat{r}(z) < \delta$, the rich consumption is decreasing, which means that they consume out of their accumulated saving. In any case, the borrowing constraint that they face does not bind.

This analysis shows that the base case can therefore be seen as a finite horizon version of the Judd (1985) workers-capitalist framework, where the rich behave like “capitalists” and the poor like “workers”. However, in the proposed model, the poor’s behavior is not assumed like in Judd (1985), but it is their optimal response to their environment. I will therefore use the model for two scenarios to study optimal taxation. In the first one, optimal taxation does not modify the poor individual’s behavior (see section 4.1). In the second scenario, the poor individual’s behavior is modified in response to tax incentives (see section 4.2).

When $\frac{1}{\lambda_R} = 0$ and $\frac{1}{\lambda_P} = 0$, the proposed model nests an infinite horizon model with two types of agents (low and high income). Both the poor and the rich have an infinite horizon and both can smooth consumption. This environment will be used as a third scenario (see section 4.3) to determine the role of income differences in optimal taxation.

Additionally, the model will also be used to study the role of life horizons. When $\frac{1}{\lambda_P} = +\infty$ and $\frac{1}{\lambda_R} > 0$, the proposed model nests a simple finite horizon model. The population is composed of one category of individuals (the rich) who have a finite lifetime. This environment will enable me to retrieve Erosa and Gervais (2002) results. When $\frac{1}{\lambda_P} = +\infty$ and $\frac{1}{\lambda_R} = 0$, the proposed model nests a simple infinite horizon model. The population is composed of one category of rich immortal individuals. In the steady state, $F'_K(t)(1 - T_k(t)) = \delta$. This environment will enable me to retrieve the Chamley (1985, 1986) and Judd (1985, 1987) results.

3.3. The government

Optimal capital and labor income tax rates are chosen to finance the exogenous government spending. The government cannot use lump-sum taxation but capital and labor income taxation is not restricted in so far as they are age-dependent. There are no transfers programs, the redistribution effects are a consequence of the optimal taxation pattern.

3.3.1. Objective

In a finite horizon model, the government has an incentive to levy a one-time tax on the wealth of agents alive. By doing that, the government could eliminate distortions and reach a first best Pareto optimum. However, such a policy would have two major inconveniences. First it could be considered unfair that generations currently alive pay all taxes while the future generations benefit from the elimination of taxation. So, it is reasonable to assume that the government sets an objective that takes into consideration the well-being of current generations as well as future generations. As a consequence, I distinguish between two elements in the government's objective. The first element takes into consideration the well-being of agents from each cohort born in $s < t$ (t is the initial date of the fiscal plan). It is equal to the integral between $-\infty$ and t of the individual well-being of each cohort alive on date t . The second element takes into consideration the well-being of an agent from each cohort born in $s > t$. It is equal to the integral from t to $+\infty$ of the individual well-being of each cohort to be born after date t . The extent to which the government wishes to redistribute welfare from current to future generations is given by the intergenerational discount factor, ρ . Second, the policy would be time inconsistent. In the social welfare function presented below, I eliminate time inconsistency related to social preferences.¹⁷ The well-being of agents alive and to be born is discounted at the same discount rate ρ and with respect to their date of birth. Agents alive and to be born are therefore treated symmetrically in the government's criteria. Furthermore, I assume that the government puts the same weight on the well-being of the poor and the rich, therefore the government's objective is:

$$SW_1(t) = \left[\int_{-\infty}^t e^{-\rho(s-t)} IW_1^R(s,t) ds + \int_t^{+\infty} e^{-\rho(s-t)} IW_1^R(s,s) ds \right] \\ + \left[\int_{-\infty}^t e^{-\rho(s-t)} IW^P(s,t) ds + \int_t^{+\infty} e^{-\rho(s-t)} IW^P(s,s) ds \right]$$

with individual well-being functions IW simply reflecting the intertemporal utility of each category of agents.

The well-being function for an individual alive on date t is

$$IW^i(s,t) = \int_t^{+\infty} e^{-\left(\delta + \frac{1}{q_i}\right)(z-s)} U[C^i(s,z), 1 - N^i(s,z)] dz$$

For an individual to be “born” on a date $s > t$, the well-being function is

$$IW^i(s,s) = \int_s^{+\infty} e^{-\left(\delta + \frac{1}{q_i}\right)(z-s)} U[C^i(s,z), 1 - N^i(s,z)] dz$$

3.3.2. The primal approach

According to the primal approach, I reformulate the government’s problem of choosing optimal taxes as a problem of choosing optimal allocations, called the “Ramsey problem”. In the usual formulation of the problem, the optimal allocations satisfy the “implementability” constraint and the “feasibility” constraint. In this paper, the optimal allocations also need to satisfy the additional borrowing constraint. The implementability constraints are simply the intertemporal budget constraints of respectively the rich and the poor (when they annuitize). Each intertemporal budget constraint is re-written using the optimality conditions of respectively the rich and the poor to replace after-tax prices by allocations. In addition, for each category of individuals, I distinguish between two types of implementability constraints. The first is faced by an individual born before t , the initial date of the plan. It takes into consideration $W(s,t)$ the wealth accumulated by the individual since birth. The second is faced by an individual born after the initial date of the plan. It takes into consideration $W(s,s)$ wealth at the time of his birth, which by assumption equals zero. The feasibility constraint is the aggregate market clearing condition. The government budget constraint is satisfied by Walras law.

Definition An allocation $\{C(s, z), N(s, z)\}_{s=-\infty}^{+\infty}, K(z)\}_{z=t}^{+\infty}$, is “implementable” for the government if:

1) $\{C^i(s, z), N^i(s, z), W^i(s, z)\}_{s=-\infty}^{+\infty}\}_{z=t}^{+\infty}$ satisfies the no-borrowing constraint:

$$W^i(s, z) \geq 0$$

2) $\{C^i(s, z), N^i(s, z), W^i(s, z)\}_{s=-\infty}^{+\infty}\}_{z=t}^{+\infty}$ satisfies the implementability constraint:

For an agent born on a date $s < t$:

$$\int_t^{+\infty} e^{-(\delta+\frac{1}{\lambda_i})(z-s)} (U_{C^i(s,z)} C^i(s, z) + U_{N^i(s,z)} N^i(s, z)) dz = U_{C^i(s,t)} W^i(s, t)$$

For an agent born on a date $s > t$:

$$\int_s^{+\infty} e^{-(\delta+\frac{1}{\lambda_i})(z-s)} (U_{C^i(s,z)} C^i(s, z) + U_{N^i(s,z)} N^i(s, z)) dz = U_{C^i(s,s)} W^i(s, s)$$

if $W^i(s, z) > 0$.

If $W^P(s, z) = 0$ the poor’s implementability constraint is replaced by the following static constraint:¹⁸

$$C^P(s, z) = -\frac{U_{N^P(s,z)}}{U_{C^P(s,z)}} N^P(s, z)$$

3) $\{C(s, z), N(s, z)\}_{s=-\infty}^{+\infty}, K(z)\}_{z=t}^{+\infty}$ satisfies the feasibility constraint:

$$\dot{K}(z) = (F'_K(z) - d)K(z) + F'_{EN}(z)E(z)N(z) - C(z) - G(z)$$

For convenience, using the primal approach, individual well-being can be re-written to include the implementability constraint. Given $e^{-(\delta+\frac{1}{\lambda_i})(z-s)} \chi^i(s)$, the multiplier associated with the implementability constraint of the generation born at time s , the individual well-being function for an individual of type i born on date $s < t$ becomes:¹⁹

$$IW^i(s, t) = \int_t^{+\infty} e^{-(\delta+\frac{1}{\lambda_i})(z-s)} [\mathcal{U}(C^i(s, z), 1 - N^i(s, z))] dz$$

where:

$$v(C^i(s, z), 1 - N^i(s, z)) = \left[\begin{array}{c} (U(C^i(s, z), 1 - N^i(s, z))) \\ + \chi^i(s) \left(\begin{array}{c} U_{C^i(s, z)} C^i(s, z) + U_{N^i(s, z)} N^i(s, z) \\ - e^{-(\delta + \frac{1}{\lambda_R})(z-s)} U_{C^i(s, t)} W^i(s, t) \end{array} \right) \end{array} \right]$$

For an individual to be born on a date $s > t$ the individual well-being function is:

$$IW^i(s, s) = \int_s^{+\infty} e^{-(\delta + \frac{1}{\lambda_R})(z-s)} [v(C^i(s, z), 1 - N^i(s, z))] dz$$

where:

$$v(C^i(s, z), 1 - N^i(s, z)) = \left[\begin{array}{c} (U(C^i(s, z), 1 - N^i(s, z))) \\ + \chi^i(s) \left(\begin{array}{c} U_{C^i(s, z)} C^i(s, z) + U_{N^i(s, z)} N^i(s, z) \end{array} \right) \end{array} \right]$$

since $W^i(s, s) = 0$.

Using the definitions of the individual welfare constraints, and switching integrals, I rewrite the government's objective as:

$$SW(t) = \int_t^{+\infty} e^{-\rho(z-t)} \left\{ \int_{s=-\infty}^z e^{\rho(z-s)} \left[\begin{array}{c} e^{-(\delta + \frac{1}{\lambda_R})(z-s)} v(C^R(s, z), 1 - N^R(s, z)) \\ + e^{-(\delta + \frac{1}{\lambda_P})(z-s)} v(C^P(s, z), 1 - N^P(s, z)) \end{array} \right] ds \right\} dz$$

for $z = [t; +\infty[$ and $\delta \leq \rho < \delta + \frac{1}{\lambda_R}$.

To simplify and make the discussion more intuitive, I set $t = 0$ and think in terms of age $n = z - s$ and time z . Then, the government's objective becomes:

$$SW(0) = \int_0^{+\infty} e^{-\rho z} \left\{ \int_0^{+\infty} e^{(\rho - \delta)n} \left[\begin{array}{c} e^{-\frac{1}{\lambda_R}n} v(C^R(z-n, z), 1 - N^R(z-n, z)) \\ + e^{-\frac{1}{\lambda_P}n} v(C^P(z-n, z), 1 - N^P(z-n, z)) \end{array} \right] dn \right\} dz \quad (6)$$

3.3.3. Solution

Solution to the Ramsey Problem:

The government maximizes the social welfare criteria (6) subject to the borrowing constraint and the feasibility constraints (the implementability constraint is included in the objective), given $K(0)$ and with $K(z) \geq 0$ for all z . The problem can be divided into 2 sub-problems. First, the government solves a static problem: On a date $z = [0; +\infty)$, the government optimally allocates a level of aggregate consumption and labor between individuals of all ages (alive and to be born), in order to

maximize their instantaneous utility. Second, the government solves a dynamic problem: The government chooses an optimal allocation path $\{C(z), N(z), K(z)\}_{z=0}^{+\infty}$, that maximizes its objective subject to the feasibility constraints (given that on all dates $z > 0$, the aggregate consumption and leisure allocations are optimally spread across ages).

The static problem:

$$V(C(z), 1 - N(z)) = \max_{\{C(z-n, z), N(z-n, z)\}_{n=0}^{+\infty}} \int_0^{+\infty} \left[e^{-\frac{1}{\lambda_R}n} \mathcal{U}(C^R(z-n, z), 1 - N^R(z-n, z)) + e^{-\frac{1}{\lambda_P}n} \mathcal{U}(C^P(z-n, z), 1 - N^P(z-n, z)) \right] e^{(\rho-\delta)n} dn$$

st.:

$$C(z) = \int_0^{+\infty} \left[\frac{1}{\lambda_P + \lambda_R} e^{-\frac{1}{\lambda_R}n} C^R(z-n, z) + \frac{1}{\lambda_P + \lambda_R} e^{-\frac{1}{\lambda_P}n} C^P(z-n, z) \right] dn \quad (7)$$

$$\tilde{N}(z) = \int_0^{+\infty} \left[\frac{1}{\lambda_P + \lambda_R} e^{-\frac{1}{\lambda_R}n} \tilde{N}^R(z-n, z) + \frac{1}{\lambda_P + \lambda_R} e^{-\frac{1}{\lambda_P}n} \tilde{N}^P(z-n, z) \right] dn \quad (8)$$

$$W^i(z-n, z) \geq 0 \quad (9)$$

The static problem is solved in a standard way. Given $\psi_1(z)$, $\psi_2(z)$, and $\psi_3^i(z)$, the multipliers associated respectively with (7), (8), and (9), the optimality conditions for all n and $z > 0$ are:

$$\left(e^{-\frac{1}{\lambda_R}n} \mathcal{U}_{C^R(z-n, z)} + e^{-\frac{1}{\lambda_P}n} \mathcal{U}_{C^P(z-n, z)} \right) e^{(\rho-\delta)n} = \frac{\psi_1(z)}{\lambda_P + \lambda_R} \left(e^{-\frac{1}{\lambda_R}n} + e^{-\frac{1}{\lambda_P}n} \right)$$

$$\left(e^{-\frac{1}{\lambda_R}n} \mathcal{U}_{N^R(z-n, z)} + e^{-\frac{1}{\lambda_P}n} \mathcal{U}_{N^P(z-n, z)} \right) e^{(\rho-\delta)n} = -\frac{\psi_2(z)}{\lambda_P + \lambda_R} \left(e^{-\frac{1}{\lambda_R}n} E^P(n) + e^{-\frac{1}{\lambda_P}n} E^R(n) \right)$$

$$\psi_3^i(z) W^i(s, z) = 0$$

The dynamic problem:

$$\max_{\{C(z), N(z), K(z)\}_{z=0}^{+\infty}} \int_{z=0}^{+\infty} e^{-\rho z} V(C(z), 1 - N(z)) dz$$

st.:

$$\dot{K}(z) = (F'_K(z) - d)K(z) + F'_{EN}(z)E(z)N(z) - C(z) - G(z)$$

with $K(0)$ given and $K(z) \geq 0$ for all z .

I use the result by Benveniste and Scheinkman (1979). If the $V(\cdot)$ function is strictly concave,²⁰ it can be considered as the value function of a dynamic problem in which z represents time. Then, $V(\cdot)$ is continuously differentiable in $C(z)$ and $N(z)$, and at the optimum:

$$V_{C(z)} = U_{C^R(z,z)} + U_{C^P(z,z)}$$

$$V_{N(z)} = U_{N^R(z,z)} + U_{N^P(z,z)}$$

for $z > 0$.

Given $\psi(z)$ the multiplier associated with the feasibility constraint, the necessary conditions of the dynamic problem are therefore (for $z > 0$):

$$V_{C(z)} = \psi(z)$$

$$V_{N(z)} = -\psi(z)E(z)\omega(z)$$

$$\dot{\psi}(z) = (\rho - r(z))\psi(z)$$

The implementability constraint incorporates the transversality condition for each individual. I have introduced the implementability constraint into the government's objective. Therefore, the transversality condition is satisfied at the aggregate level.

Optimal tax rates:

Once the government problem is solved, it is possible to find the expressions for the optimal capital and labor income tax rates after date zero. Optimal capital income taxation is deduced from the difference between individual and social marginal rates of substitution between current and future consumption (MRSC).

For the rich:

$$\delta - \hat{r}^R(z-n, z) - (\rho - r(z)) = \frac{\dot{U}_{C^R(z-n, z)}}{U_{C^R(z-n, z)}} - \frac{\dot{V}_{C(z-n, z)}}{V_{C(z-n, z)}} \quad (10)$$

For the poor, if they do not annuitize, capital income taxation satisfies:

$$\delta + \frac{1}{\lambda_p} - \hat{r}^P(z-n, z) - (\rho - r(z)) \geq \frac{\dot{U}_{C^P(z-n, z)}}{U_{C^P(z-n, z)}} - \frac{\dot{V}_{C(z-n, z)}}{V_{C(z-n, z)}} \quad (11)$$

If they annuitize, capital income taxation satisfies:

$$\delta - \hat{r}^P(z - n, z) - (\rho - r(z)) = \frac{\dot{U}_{C^P(z-n,z)}}{U_{C^P(z-n,z)}} - \frac{\dot{V}_{C(z-n,z)}}{V_{C(z-n,z)}}$$

Optimal labor income taxes (T_ω^P and T_ω^R) are deduced from the ratio of individual and social marginal rates of substitution between consumption and leisure (MRSL):

$$T_\omega^P(z - n, z) = 1 - \frac{\frac{U_{N^P(z-n,z)}}{U_{C^P(z-n,z)}}}{\frac{V_{N(z-n,z)}}{V_{C(z-n,z)}}}$$

$$T_\omega^R(z - n, z) = 1 - \frac{\frac{U_{N^R(z-n,z)}}{U_{C^R(z-n,z)}}}{\frac{V_{N(z-n,z)}}{V_{C(z-n,z)}}}$$

4. Results

First, the focus of this discussion is the steady state. In a finite horizon framework, in the steady state, the government allocates aggregate consumption and labor optimally between individuals of all ages n . Since choosing optimal allocations is equivalent to choosing optimal tax rates on capital and labor income, the expressions below provide steady state age-dependent tax rates. Second, the discussion concentrates on capital income taxation. The reason is that the effect of the borrowing constraints on optimal labor income taxation is found to be ambiguous. The limited access to credit markets for the poor does not necessarily result in a more progressive labor income tax. Since this result presents less interest, the proof is shown in Appendix A.

Following (10), steady state capital income taxation for the rich is deduced from:

$$r - \hat{r}^R(n) = \frac{\frac{\partial U_{C^R(n)}}{\partial n}}{U_{C^R(n)}} - \frac{\frac{\partial}{\partial n} [\gamma(n)v_{C^R(n)} + (1 - \gamma(n))v_{C^P(n)}]}{\gamma(n)v_{C^R(n)} + (1 - \gamma(n))v_{C^P(n)}} \quad (12)$$

where $\frac{\partial U_{C^R(n)}}{\partial n}$ is the individual MRSC and $\frac{\frac{\partial \gamma(n)}{\partial n} (\gamma(n) U_{C^R(n)} - (1-\gamma(n)) U_{C^P(n)})}{\gamma(n) U_{C^R(n)} + (1-\gamma(n)) U_{C^P(n)}}$ is the social MRSC. The weights of $U_{C^R(n)}$ and $U_{C^P(n)}$ in the government's optimality conditions are denoted $\gamma(n) = \frac{e^{-\frac{1}{\lambda_R} n}}{e^{-\frac{1}{\lambda_R} n} + e^{-\frac{1}{\lambda_P} n}}$ and $1 - \gamma(n)$. The variable $U_{C^R(n)}$ is re-expressed as follows:

$$U_{C^R(n)} = (1 + \chi^R + \chi^R \xi^{C^R(n)}) U_{C^R(n)}$$

with $\xi^{C^R(n)}$ the general equilibrium elasticity of the rich:

$$\xi^{C^R(n)} = \frac{U_{C^R(n)C^R(n)} C^R(n) + U_{L^R(n)C^R(n)} L^R(n)}{U_{C^R(n)}}$$

Expression (12) has several ingredients indicating why the social MRSC generally differs from the private MRSC and capital income taxation may differ from zero. These ingredients are the poor and the rich individuals' marginal utilities of consumption, $U_{C^P(n)}$ and $U_{C^R(n)}$, their horizons, λ_P and λ_R , which determine the weights $\gamma(n)$ and $1 - \gamma(n)$ and the general equilibrium elasticity $\xi^{C^R(n)}$.

When the poor do not annuitize, they face a binding borrowing constraint in the initial steady state. Following (11), capital income taxation for the poor is deduced from:

$$\frac{1}{\lambda_P} + r - \hat{r}^P(n) \geq \left(\frac{\partial U_{C^P(n)}}{\partial n} \right) - \frac{\frac{\partial}{\partial n} [\gamma(n) U_{C^R(n)} + (1-\gamma(n)) U_{C^P(n)}]}{\gamma(n) U_{C^R(n)} + (1-\gamma(n)) U_{C^P(n)}} \quad (13)$$

with $U_{C^P(n)}$ is rewritten as:

$$U_{C^P(n)} = (1 + \chi^P + \chi^P \xi^{C^P(n)}) U_{C^P(n)}$$

and $\xi^{C^P(n)}$ the general equilibrium elasticity of the poor:

$$\xi^{C^P(n)} = \frac{U_{C^P(n)C^P(n)} C^P(n) + U_{L^P(n)C^P(n)} L^P(n)}{U_{C^P(n)}}$$

The following scenarios are then considered. In section 4.1, the optimal capital tax rates are such that the poor still face a binding borrowing

constraint at the optimum ($\hat{r}^P(n) < \delta + \frac{1}{\lambda_p}$). This scenario corresponds to a finite horizon version of the “workers-capitalist” model by Judd (1985).²¹ If the borrowing constraint binds, $U_{c^p(n)} = U_{c^p(n)}$ in equations (12) and (13), equation (13) holds as an inequality. In section 4.2, the optimal tax scheme relieves the poor from the binding borrowing limit at the optimum ($\hat{r}^P(n) > \delta + \frac{1}{\lambda_p}$). If, at the optimum, the borrowing constraint does not bind, equation (13) holds as an equality.

When both the poor and the rich annuitize, borrowing constraints are non-binding in the initial steady state. Optimal capital income tax for the rich is given by (12). Optimal taxation for the poor is given by:

$$r(n) - \hat{r}^P(n) = \left(\frac{\frac{\partial U_{c^p(n)}}{\partial n}}{U_{c^p(n)}} \right) - \frac{\frac{\partial}{\partial n} [\gamma(n)U_{c^R(n)} + (1-\gamma(n))U_{c^P(n)}]}{\gamma(n)U_{c^R(n)} + (1-\gamma(n))U_{c^P(n)}}$$

I study that case in section 4.3.

4.1. Finite horizon “workers-capitalist” scenario

Proposition 1 and its corollary clarify the conditions for non-zero capital income taxation for the rich. Proposition 2 studies optimal capital income taxes faced by the poor relatively to the rich and discusses the progressivity of the optimal taxation scheme.

Proposition 1: Optimal age-dependent capital income taxation for the rich differs from zero, if one of the following sufficient (but not necessary) conditions is satisfied:

P1: $U_{c^p(n)} \neq U_{c^R(n)}$.

P2: $\lambda_p \neq \lambda_R$ and preferences are non-logarithmic.

P3: Preferences are not additively separable between consumption and leisure.

Proof

–To show that (P1) is sufficient for optimal capital income taxation to be different from zero, I show that equation (12) differs from zero when (P1) is satisfied but (P2) and (P3) are not. I set $U_{c^p(n)} \neq U_{c^R(n)}$ $\lambda_p = \lambda_R$, and logarithmic preferences ($\xi^{c^R(n)}$ is constant and equal to –1) Condition (P3) is not satisfied since logarithmic preferences are additively separable between consumption and leisure. Equation (12) simplifies and capital income taxation is deduced from:

$$r - \hat{r}^R(n) = \frac{\frac{\partial U_{C^R}(n)}{\partial n}}{U_{C^R}(n)} - \frac{\frac{\partial U_{C^R}(n)}{\partial n} + \frac{\partial U_{C^P}(n)}{\partial n}}{U_{C^R}(n) + U_{C^P}(n)} \quad (14)$$

This equation shows that the private MRSC is different from the social MRSC. Optimal capital income taxation is therefore different from zero.

The Proof that (P1) is sufficient for optimal capital income taxation to be different from zero also proves that (P2), and non-additively separable preferences (P3), are non necessary to obtain $T_k^R(n) \neq 0$.

–To show that (P2) is a sufficient for optimal capital income taxation to be different from zero, I show that (12) differs from zero when (P2) is satisfied but (P1) and (P3) are not. Setting $\lambda_p \neq \lambda_R$ and non-logarithmic preferences, $U_{C^P}(n) = U_{C^R}(n)$ and additively separable preferences ($\xi^{C^R(n)}$ constant and equals $-\sigma^{C^R} \neq -1$), equation (12) simplifies and capital income taxation is deduced from:

$$r - \hat{r}^R(n) = \frac{\frac{\partial \gamma(n)}{\partial n} \chi^R (1 - \sigma^{C^R})}{\gamma(n) \chi^R (1 - \sigma^{C^R}) + 1} \quad (15)$$

This equation shows that optimal capital income taxation differs from zero. The social MRSC depends on the poor and the rich respective horizons, which determine $\gamma(n)$. However, the different horizons of the poor and the rich matter only when the elasticity of substitution of consumption differs from one.

The proof that (P2) is sufficient for optimal capital income taxation to be different from zero also proves that (P1) and (P3) are not necessary to obtain $T_k^R(n) \neq 0$.

–To show that (P3) is a sufficient condition for non-zero optimal capital income taxation, I show that (12) differs from zero when (P3) is satisfied but (P1) and (P2) are not. Setting non-additively separable preferences ($\xi^{C^R(n)}$ non constant), $U_{C^P}(n) = U_{C^R}(n)$ and $\lambda_p = \lambda_R$, equation (12) simplifies and capital income taxation is deduced from:

$$r - \hat{r}^R(n) = \frac{\chi^R \frac{\partial \xi^{C^R(n)}}{\partial n}}{\chi^R (1 + \xi^{C^R(n)}) + 2} \quad (16)$$

Optimal capital income taxation is therefore different from zero. It depends on the form of the rich individual's preferences (which determine $\xi^{C^R(n)}$). Because of the binding borrowing constraint, the form of the preferences of the poor is irrelevant since they cannot reallocate consumption across periods.

The proof that (P3) is sufficient for optimal capital income taxation to be different from zero also proves that (P1) and (P2) are not necessary to obtain $T_K(n) \neq 0$

Corollary A: Optimal capital income taxation for the rich equals zero if condition (C1) or (C2) is satisfied:

C1: $U_{C^P}(n) = U_{C^R}(n)$ preferences are separable additively between consumption and leisure ($\sigma^{C^R} \neq 1$) and $\lambda_P = \lambda_R$.

C2: $U_{C^P}(n) = U_{C^R}(n)$ and preferences are logarithmic.

Proof

–The proof that optimal capital income taxation equals zero if (C1) is satisfied can be directly deduced from (14), (15) or (16). According to equation (16), capital income taxation differs from zero when $U_{C^P}(n) = U_{C^R}(n)$ and $\lambda_P = \lambda_R$, as long as preferences are not additively separable between consumption and leisure. It is therefore straightforward that with additively separable preferences, capital income taxation equals zero. In that case, the model is reduced to a life-cycle model in which agents differ only by their age, labor choices and preferences. According to equation (16), if preferences are additively separable, then $\xi^{C^R(n)}$ is constant, $\frac{\partial}{\partial n} \xi^{C^R(n)} = 0$, and capital income taxation equals zero.

–The proof that optimal capital income taxation equals zero if (C2) is satisfied can be deduced from (15). According to (15), capital income taxation differs from zero when $\lambda_P \neq \lambda_R$, $U_{C^P}(n) = U_{C^R}(n)$ and $\xi^{C^R(n)}$ constant but different from -1 (non logarithmic preferences). If preferences are logarithmic ($\sigma^{C^R} = 1$), the numerator of (15) is zero.

In that case, capital income taxation is therefore equal to zero.

Proposition 2: When the private MRSC of the rich is larger than the social MRSC, optimal capital income taxation paid by the rich is positive and optimal capital income taxation faced by the poor equals zero. As a result, the optimal capital income tax scheme is progressive.

Proof

If:
$$\frac{\frac{\partial U_{C^R(n)}}{\partial n}}{U_{C^R(n)}} > \frac{\frac{\partial}{\partial n} \left(\gamma(n) v_{C^R(n)} + (1 - \gamma(n)) U_{C^P(n)} \right)}{\gamma(n) v_{C^R(n)} + (1 - \gamma(n)) U_{C^P(n)}}$$

From (12):

$$r - \hat{r}^R(n) > 0$$

$T_k^R(n)$ is therefore positive.

When the poor face a binding borrowing constraint, at the optimum, $T_k^P(n) = 0$. Indeed, when the no-borrowing constraint is binding, $W^P = 0$.

This result shows that contrary to Judd (1985), in a finite horizon framework, capital income taxation faced by “capitalists” generally differs from zero and that the optimal tax scheme involves redistribution. Contrary to the workers-capitalist framework, in my model, the decision made by the poor to consume and save is optimal. The poor are therefore susceptible to change their consumption saving decision if the interest rate is modified by tax policy. I investigate this scenario in the next section.

4.2. Borrowing limits loosened

In this scenario, the optimal tax rates involve a behavioral response by the poor. As a consequence, they are relieved from the binding borrowing constraint. Proposition 3 and its corollary determine the conditions for non-zero optimal capital income taxation by the rich in the steady state. Proposition 4 studies the redistribution aspect of the optimal tax scheme.

Proposition 3: Optimal age-dependent capital income taxation for the rich differs from zero, if one of the following sufficient (but not necessary) conditions is satisfied: P1, P4 or P5.

P4: $\lambda_p \neq \lambda_r$ and preferences are non logarithmic and $\chi^R(1 + \sigma^{C^R}) \neq \chi^P(1 + \sigma^{C^P})$

P5: preferences are non-additively separable and $\chi^R(1 + \frac{\partial}{\partial n} \xi^{C^R(n)}) \neq \chi^P(1 + \frac{\partial}{\partial n} \xi^{C^P(n)})$

Condition (P1) is similar to the case with no-borrowing constraints. (P4) and (P5) are new conditions.

Corollary B: Optimal capital income taxation for the rich equals zero if condition (C1) or (C3) or (C4) is satisfied.

C3: $U_{C^P}(n) = U_{C^R}(n)$ preferences are separable additively between consumption and leisure and $\chi^R(1 + \sigma^{C^R}) \neq \chi^P(1 + \sigma^{C^P})$.

C4: $U_{C^P}(n) = U_{C^R}(n)$ $\lambda_P = \lambda_R$ and $\chi^R(1 + \frac{\partial}{\partial n} \xi^{C^R(n)}) \neq \chi^P(1 + \frac{\partial}{\partial n} \xi^{C^P(n)})$

Compared to the case when the poor face a binding no-borrowing constraint, there are therefore additional cases for optimal capital income taxation to be zero. In these additional scenarios, individuals have the same optimal way to reallocate consumption over time at age n . The proofs of Proposition 3 and Corollary B are presented in Appendix B.

Proposition 4: When the private MRSC is larger than the social MRSC, optimal capital income taxation paid by the rich is positive. When $U_{C^P}(n) > U_{C^R}(n)$, optimal capital income taxation is larger for the rich than for the poor. As a result, the optimal capital income tax scheme is progressive.

Proof

If the poor are relieved from the binding borrowing constraint, from (12), if:

$$\frac{\frac{\partial U_{C^R}(n)}{\partial n}}{U_{C^R}(n)} > \frac{\frac{\partial}{\partial n} (\gamma(n) v_{C^R(n)} + (1 - \gamma(n)) v_{C^P(n)})}{\gamma(n) v_{C^R(n)} + (1 - \gamma(n)) v_{C^P(n)}}$$

then:

$$r - \hat{r}^R(n) > 0$$

$T_k^R(n)$ is therefore positive.

Furthermore, equation (13) becomes:

$$\frac{1}{\lambda_P} + r - \hat{r}^P(n) = \left(\frac{\frac{\partial U_{C^P}(n)}{\partial n}}{U_{C^P}(n)} \right) - \frac{\frac{\partial}{\partial n} [\gamma(n) v_{C^R(n)} + (1 - \gamma(n)) v_{C^P(n)}]}{\gamma(n) v_{C^R(n)} + (1 - \gamma(n)) v_{C^P(n)}} \quad (17)$$

If: $U_{C^P}(n) > U_{C^R}(n)$

and: $\frac{\frac{\partial U_{C^R}(n)}{\partial n}}{U_{C^R}(n)} > \frac{\frac{\partial}{\partial n} (\gamma(n) v_{C^R(n)} + (1 - \gamma(n)) v_{C^P(n)})}{\gamma(n) v_{C^R(n)} + (1 - \gamma(n)) v_{C^P(n)}}$

then: $r - \hat{r}^R(n) > \frac{1}{\lambda_P} + r - \hat{r}^P(n) > r - \hat{r}^P(n)$

Therefore, $T_k^R(n) > T_k^P(n)$

As they are relieved from the binding borrowing constraint, the poor achieve welfare gains related to consumption smoothing. The optimal capital income tax scheme is progressive across income levels. The term $\frac{1}{\lambda_p}$ denotes that the capital income tax (or subsidy) faced by the poor is small (or large) enough to create a saving incentive for the poor in the absence of annuitization.

The simple case of logarithmic preferences enables me to be more specific on the sign of optimal capital income taxation faced by the poor.

Corollary C: When preferences are logarithmic, $\lambda_p = \lambda_R$ and $U_{C^p}(n) > U_{C^r}(n)$ it is optimal to tax capital income by the rich and subsidize the poor's capital income.

Proof

When preferences are logarithmic, $\lambda_p = \lambda_R$ and $U_{C^p}(n) > U_{C^r}(n)$

$$\frac{\frac{\partial U_{C^r}(n)}{\partial n}}{U_{C^r}(n)} > \frac{\frac{\partial U_{C^r}(n)}{\partial n} + \frac{\partial U_{C^p}(n)}{\partial n}}{U_{C^r}(n) + U_{C^p}(n)} > \frac{\frac{\partial U_{C^p}(n)}{\partial n}}{U_{C^p}(n)}$$

then, from (12) and (17):

$$\begin{aligned} r - \hat{r}^R(n) &> 0 \\ r - \hat{r}^P(n) &< -\frac{1}{\lambda_p} \end{aligned}$$

Therefore, $T_k^R(n)$ is positive and $T_k^P(n)$ is negative.

The subsidy to the poor has to be large enough at any age to create a saving incentive for the poor in the absence of annuitization.

When the poor face a binding constraint in the initial steady state, there are generally two optimal capital income tax regimes, one for the poor, one for the rich. Progressive capital income taxation has the potential to relieve the poor from binding borrowing limits. In that case, social welfare increases as the result of consumption smoothing benefits for the poor.

The conditions for zero capital income taxation are more restrictive than in the simple life-cycle model. Proposition 1 and 3 contrast with Erosa and Gervais (2002) for whom non-additively separable preferences

are necessary to obtain $T_k(n) \neq 0$. Their corollaries also contrasts with Erosa and Gervais (2002) for whom additively separable preferences alone are sufficient to obtain $T_k(n) = 0$. Because of the presence of the poor, capital income taxation plays a redistributive role across income categories, which influences the tax rate faced by the rich and renders the optimal capital income tax scheme progressive.

4.3. Non-binding borrowing constraints

In the case of non-binding borrowing constraints, Proposition 3 and Corollary B remain valid. In this scenario, the rich annuitize and they never face binding borrowing constraints. Optimal capital income taxation faced by the rich takes into consideration that the poor can reallocate consumption over their lifetime. The theoretical conditions for optimal capital income taxation faced by the rich to be zero are therefore the same as in the scenario with loosened borrowing constraints. The reasons are as follows. The rich have the same consumption and saving behavior than in the previous case. The poor and the rich differ only by their differences in income. The difference in income levels between the poor and the rich is sufficient to create a gap between the rich and the poor marginal utilities and a thereby a redistribution motive between individuals with separate life-cycles.

The simple case of logarithmic preferences enables me to compare optimal taxation in the case the poor never face binding borrowing constraints with the case when they are relieved from an existing binding borrowing constraints. Let the subscript B denote a binding borrowing constraint and the subscript NB denote a non-binding borrowing constraint.

Proposition 5: When preferences are logarithmic, $\lambda_p = \lambda_R$ and $U_{C^p}(n) > U_{C^R}(n)$ it is optimal to subsidize the poor less in case the poor face a non-binding borrowing constraint rather than a binding borrowing constraint

Proof

When preferences are logarithmic, and $\lambda_p = \lambda_R$.

$$r - \hat{r}_{NB}^P(n) = \frac{\partial U_{C^p}(n)}{\partial n} - \frac{\partial U_{C^R}(n)}{\partial n} + \frac{\partial U_{C^p}(n)}{\partial n} \\ U_{C^p}(n) \quad U_{C^R}(n) + U_{C^p}(n)$$

If $U_{c^p}(n) > U_{c^r}(n)$ $T_{kNB}^P < 0$. Then, from (17):

$$0 > \frac{1}{\lambda_p} + r - \hat{r}_b^P(n) = r - \hat{r}_{NB}^P(n)$$

Therefore, $T_{kNB}^P > T_{NB}^P$. The subsidy to the poor is smaller when the borrowing constraint is non-binding. The difference between the two scenarios reflects the fact that when the poor annuitize, a smaller tax subsidy stimulates capital accumulation. When they do not annuitize, a larger subsidy at all ages is necessary to compensate for the absence of annuities, relieve them from the binding borrowing constraint and stimulate capital accumulation.²²

The conditions for optimal capital income taxation faced by the rich to be different from zero are similar whether borrowing constraints faced by the poor bind or not in the initial steady state.²³ However, the conditions for zero capital income taxation remain more restrictive than in the simple life-cycle model by Erosa and Gervais (2002). They are also more restrictive than in the model by Conesa *et al.* (2007, 2009), which includes uninsurable income shocks and permanent productivity differences of households. In addition, the optimal capital income tax system is generally progressive. The case of logarithmic preferences enables to show that the subsidy to the poor is smaller when the borrowing constraint is non-binding in the initial steady state. Welfare gains can be achieved by relieving the poor from the borrowing limit using relatively large subsidies.

4.4. Different versus same life horizons

From Propositions 1, 3 and Corollaries A and B, we first learn that, in the case of non logarithmic preferences, $\lambda_p \neq \lambda_r$ guarantees that capital income taxation faced by the rich differs from zero in the long run. In this case, individuals who have different MRSC also have different discount rates. If the social welfare function takes into consideration the welfare of the poor and the rich, it reflects the fact that poor and rich individuals discount the future at different rates. This creates a gap between individual and social MRSC, resulting in an optimal tax rate on capital income which differs from zero. Therefore the conditions for

non-zero capital income taxation in the steady state are reinforced when different life horizons are taken into consideration.

Second, we learn that when the poor and the rich have logarithmic preferences and $U_{C^P}(n) = U_{C^R}(n)$, individuals horizons are irrelevant. Their MRSC are constant and equal to one. If $U_{C^P}(n) = U_{C^R}(n)$, poor and rich individuals are willing to reallocate consumption over ages the same way. As a result, their horizons and therefore their weights in the social welfare function are irrelevant since there is no difference between the private and the social MRSC. In that case, capital income taxation equals zero (see (15)).

By contrast, if $U_{C^P}(n) \neq U_{C^R}(n)$ their horizons matter. To have some insight on their roles for redistribution, I study the case of logarithmic preferences. The next proposition studies the effect on optimal capital income taxation of an increase in the life horizon of the rich relatively to the life horizon of the poor.

Proposition 6: When preferences are logarithmic and $U_{C^P}(n) > U_{C^R}(n)$, the longer the life horizon of the rich relatively to the poor, the lower the capital income tax paid by the rich. It remains optimal to subsidize the poor's capital income. The optimal taxation scheme is progressive.

Proof

When the proportion of the rich in the total population becomes extremely large: $\gamma(n) \mapsto 1$ then:

$$\frac{\frac{\partial}{\partial n} [\gamma(n)U_{C^R(n)} + (1-\gamma(n))U_{C^P(n)}]}{\gamma(n)U_{C^R(n)} + (1-\gamma(n))U_{C^P(n)}} \mapsto \frac{\frac{\partial U_{C^R(n)}}{\partial n}}{U_{C^R(n)}}$$

From (12): $r(n) - \hat{r}^R(n) \mapsto 0$

Therefore, the longer the rich horizon relatively to the poor horizon, the closer to zero capital income taxation for the rich.

Furthermore, if the poor do not annuitize, optimal capital income taxation for the poor is given by:

$$\frac{1}{\lambda_P} + r(n) - \hat{r}^P(n) \mapsto \frac{\frac{\partial U_{C^P(n)}}{\partial n}}{U_{C^P(n)}} - \frac{\frac{\partial U_{C^R(n)}}{\partial n}}{U_{C^R(n)}}$$

and if the poor annuitize:

$$r(n) - \hat{r}^P(n) \mapsto \frac{\frac{\partial U_{C^P}(n)}{\partial n}}{U_{C^P}(n)} - \frac{\frac{\partial U_{C^R}(n)}{\partial n}}{U_{C^R}(n)}$$

With $U_{C^P}(n) > U_{C^R}(n)$ the right hand side is negative; therefore, the optimal capital taxation scheme involves a subsidy to the poor.

Even with a relatively small weight for the poor individual welfare in the social welfare function, there are some social welfare gains to subsidize the poor's return on wealth.

Limit case: Erosa and Gervais (2002)

It is possible to retrieve Erosa and Gervais (2002) results by setting $\gamma(n) = 100\%$. This results in eliminating the poor from the economy. The model becomes a continuous time life-cycle model with only rich agents. Capital income taxation is then given by:

$$r(n) - \hat{r}^R(n) = \frac{\frac{\partial U_{C^R}(n)}{\partial n}}{U_{C^R}(n)} - \frac{\frac{\partial U_{C^R}(n)}{\partial n}}{U_{C^R}(n)}$$

which is equivalent to:

$$r(n) - \hat{r}^R(n) = - \frac{\frac{\partial \xi^{C^R}(n)}{\partial n}}{1 + \chi^R + \chi^R \xi^{C^R}(n)}$$

Similarly to Erosa and Gervais (2002), when preferences are additively separable, $\xi^{C^R}(n) = -1$ and $\frac{\partial \xi^{C^R}(n)}{\partial n} = 0$. Therefore, the above expression equals zero. In the case of one unique life-cycle, the presence of borrowing constraints is irrelevant for optimal capital income taxation. The representative agent accumulates wealth and his borrowing constraint is non-binding. Therefore, it has no influence on his consumption allocation across ages.

Same infinite horizon: Judd 1985

The model enables me to retrieve the results by Judd (1985) in the "workers-capitalist" framework. When both the poor and the rich have the same infinite lifetime, $\frac{1}{\lambda_p} = 0$ and $\frac{1}{\lambda_r} = 0$. As a result, $\gamma(n) = 1 - \gamma(n) = 50\%$. The poor and the rich have the same weight in the social welfare function. In addition, in an infinite horizon model, $\xi^{C(n)}$ is by definition

constant across ages. Consumption is constant in the steady state. As a result, capital income taxation faced by the rich equals zero.

5. Conclusion

The standard results of the Ramsey optimal taxation literature, derived in an infinite horizon framework have been challenged by recent life-cycle models. I extend this literature with a new theoretical contribution. In a new overlapping generations model, individuals face different income levels and life expectancies. As a consequence, they make different annuitization choices, which in the presence of borrowing constraints leads to different consumption and saving behaviors. In that framework, I find that the conditions for zero optimal capital income taxation are more restrictive than in the simple life-cycle model. Optimal capital income taxation generally differs from zero in the long term even when preferences are additively separable. I also find that the optimal capital income tax system is generally progressive in the long run. Furthermore, the model enables to disentangle the roles of finite horizons, income differences and borrowing constraints. I find that in a finite horizon framework, differences in income alone can lead to a non-zero optimal capital income tax for the rich and a progressive tax scheme. Capital income taxation faced by the rich is indirectly influenced by borrowing limits through their effect on the poor's behavior. Borrowing limits directly determine optimal capital income taxation for the poor. If borrowing limits bind, it is optimal to subsidize the poor's capital income at a relatively larger rate than if they do not bind, in order to loosen them. The conditions for non-zero capital income taxation in the steady state are generally reinforced when different life horizons are taken into consideration.

On one hand, those results are consistent with the observation that capital income taxation remains positive in Latin American countries. On the other hand, the results call into question the degree of progressivity of tax systems in Latin America. For example, the Brazilian tax system is overall regressive. The theoretical results of this

paper are unclear whether the existence of inequalities and borrowing limits appeal for progressivity in labor income taxation. By contrast, the various scenarios considered suggest that the tax system should generally include a progressive capital income tax scheme as it generates welfare gains for the poor from consumption smoothing and redistribution. Policy recommendations need to be taken with caution and a natural extension of the paper would be to provide country specific numerical simulations of the model.

6. References

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7. Appendix

Appendix A

The labor income taxes are written in the steady state:

$$T_{\omega}^P(n) = 1 - \frac{\frac{U_{N^P(n)}}{U_{C^P(n)}}}{\frac{\gamma_E(n) \mathcal{D}_{N^R(n)}^{+(1-\gamma_E(n))} \mathcal{D}_{N^P(n)}}{\gamma(n) \mathcal{D}_{C^R(n)}^{+(1-\gamma(n))} \mathcal{D}_{C^P(n)}}} \quad (18)$$

$$T_{\omega}^R(n) = 1 - \frac{\frac{U_{N^R(n)}}{U_{C^R(n)}}}{\frac{\gamma_E(n) \mathcal{D}_{N^R(n)}^{+(1-\gamma_E(n))} \mathcal{D}_{N^P(n)} / E(n)}{\gamma(n) \mathcal{D}_{C^R(n)}^{+(1-\gamma(n))} \mathcal{D}_{C^P(n)}}} \quad (19)$$

where: $\gamma_E(n) = \frac{e^{-\frac{1}{\lambda_R}n}}{e^{-\frac{1}{\lambda_R}n} E^h(n) + e^{-\frac{1}{\lambda_P}n} E^P(n)}$.

From (18) and (19), it is straightforward that if:

$$\frac{U_{N^P(n)}}{U_{C^P(n)}} > \frac{U_{N^R(n)}}{U_{C^R(n)}} \quad (20)$$

or equivalently if:

$$\frac{U_{N^P(n)}}{U_{N^R(n)}} > \frac{U_{C^P(n)}}{U_{C^R(n)}}$$

then:

$$T_{\omega}^P(n) < T_{\omega}^R(n)$$

If (20) holds, labor income taxation is progressive across income categories.

Consumption is a function of human and financial wealth. Two main elements influence human wealth: lifetime expectancies and income profiles. As a consequence, other things equal, the rich have a larger human wealth than the poor. Asset accumulation is influenced by borrowing constraints. Therefore, given some income profiles for the poor and the rich, the more stringent the borrowing constraint on the poor compared to the rich, the larger the difference between their consumptions. As a result, the larger $\frac{U_{C^P}(n)}{U_{C^R}(n)}$. However, the counter effect is the income effect on labor supply which raises the ratio of marginal labor disutility on the left hand side of the inequality. As a result, the effect of the borrowing constraints on optimal labor income taxation is ambiguous and labor income taxation can be progressive if the former effect is larger than the latter or regressive otherwise.

Appendix B

Proof of proposition 3:

–To show that condition (P1) is sufficient but not necessary for capital income taxation to differ from zero for the rich, I show that (12) differ from zero when (P1) is satisfied but (P2) and (P3) are not. Since condition (P4) and (P5) includes condition (P2) and (P3), if (P2) and (P3) are not satisfied, (P4) and (P5) are not satisfied either. –Following the same logic, to show that condition (P4) is sufficient but not necessary for capital income taxation to differ from zero for the rich, I show that (12) differ from zero when (P4) is satisfied but (P1) and (P5) are not. Setting $\lambda_P \neq \lambda_R$ and non logarithmic preferences, $U_{C^P}(n) = U_{C^R}(n)$ and additively separable preferences ($\xi^{C^R(n)}$ and $\xi^{C^P(n)}$ are constant and respectively equal to $-\sigma^{C^R} \neq -1$ and $-\sigma^{C^P} \neq -1$), equation (12) becomes:

$$\frac{\frac{\partial}{\partial n} \gamma(n) \left[\chi^R (1 + \sigma^{C^R}) - \chi^P (1 + \sigma^{C^P}) \right]}{\gamma(n) (1 + \chi^R + \chi^R \sigma^{C^R}) + (1 - \gamma(n)) (1 + \chi^P + \chi^P \sigma^{C^P})} \quad (21)$$

This expression differs from zero unless $\lambda_p = \lambda_R$ or $\chi^R(1 + \sigma^{C^R}) = \chi^P(1 + \sigma^{C^P})$, which is the case when preferences are logarithmic.

–To show that conditions (P5) is sufficient but not necessary for capital income taxation to differ from zero, I show that (12) differs from zero when (P5) is satisfied but (P1) and (P4) are not. Starting from equation (12), the proof that optimal capital income taxation for the rich differs from zero is straightforward. When preferences are non additively separable and $\chi^R(1 + \frac{\partial}{\partial n} \xi^{C^R(n)}) \neq \chi^P(1 + \frac{\partial}{\partial n} \xi^{C^P(n)})$, $U_{C^R}(n) = U_{C^R}(n)$ and $\lambda_p = \lambda_R$, (12) becomes:

$$\frac{\chi^R(1 + \frac{\partial}{\partial n} \xi^{C^R(n)}) - \chi^P(1 + \frac{\partial}{\partial n} \xi^{C^P(n)})}{(1 + \chi^R + \chi^R \xi^{C^R(n)}) + (1 + \chi^P + \chi^P \xi^{C^P(n)})} \quad (22)$$

This expression differs from zero unless $\chi^R(1 + \frac{\partial}{\partial n} \xi^{C^R(n)}) \neq \chi^P(1 + \frac{\partial}{\partial n} \xi^{C^P(n)})$, which is the case when preferences are additively separable

Proof of the Corollary B:

–The proof that optimal capital income taxation equals zero if (C1) is satisfied can be directly deduced from (21). This expression equals zero if $\lambda_p = \lambda_R$.

–The proof that optimal capital income taxation equals zero if (C3) is satisfied can be directly deduced from (21). This expression equals zero if $\chi^R(1 + \sigma^{C^R}) = \chi^P(1 + \sigma^{C^P})$, which is the case when preferences are logarithmic.

–The proof that optimal capital income taxation equals zero if (C4) is satisfied can be directly deduced from (22). This expression equals zero if $\chi^R(1 + \frac{\partial}{\partial n} \xi^{C^R(n)}) \neq \chi^P(1 + \frac{\partial}{\partial n} \xi^{C^P(n)})$.

8. Notes

- 1 In this framework, taxation is defined as progressive if the tax rate faced by the rich is larger than the tax rate faced by the poor.
- 2 The study is based on data from year 2005.
- 3 Despite the fact that the model does not include consumption taxation, but only age-dependant capital and labor income taxation, the theoretical results are useful to analyze fiscal policy in Latin America. Indeed, according

- to Erosa and Gervais (2002) consumption taxation can be eliminated from the study of optimal age-dependent taxation without loss of generality.
- 4 See Auerbach and Hines (2001) for an overview of the optimal taxation theory in closed economies, and Slemrod (2007) for the open economy.
 - 5 While rich individuals may die holding a positive wealth, new generations are born with no financial wealth. The turnover of generations causes a drag on aggregate consumption growth for the rich and as a result, the economy reaches a positive aggregate stock of capital in the steady state only if their individual consumption grows and rich individuals accumulate wealth. See section 3.2.
 - 6 The portion of savings related to precautionary saving is not directly observable and recent findings suggest that it has a negligible macroeconomic effect. In an infinite horizon model, Aiyagari (1994) finds that precautionary saving accounts for 1-2% of aggregate wealth. Feigenbaum (2007) shows that in a finite horizon general equilibrium model, the contribution of precautionary saving to the capital stock is negligible in the presence of no-borrowing constraints. Gourinchas and Parker (2001) find that this precautionary saving accounts for 45-65% of aggregate wealth but their result is obtained in a partial equilibrium framework with a fixed interest rate. Feigenbaum (2007) also shows that precautionary saving contributes more significantly to the aggregate capital stock when borrowing constraints are endogenous.
 - 7 See literature review for more details.
 - 8 Infinite horizon can arise due to an infinite lifetime for the representative agent or an infinite horizon for altruistic families.
 - 9 Other recent contributions include other features such as human capital or public good provision in the OLG framework. They find that capital income taxation is generally positive in the long run (Pirttila and Tuomala [2001], Echevaria and Iza [2000], Bovenberg and Van Ewijk [1997]). The positive approach to taxation also suggests that capital income taxation should differ from zero when markets are incomplete (Hubbard and Judd (1986), Imrohoroglu (1998)). Lozachmoeur (2006) focuses on labor income taxation in an OLG model with borrowing constraints.
 - 10 Time until death has an exponential distribution, therefore:

$$E \int_t^{+\infty} e^{-\delta(z-t)} U[C^i(s, z), L^i(s, z)] dz$$

is equivalent to:

$$\int_0^{+\infty} e^{-\left(\delta+\frac{1}{\lambda_i}\right)(z-t)} U[C^i(s, z), L^i(s, z)] dz$$

- 11 In the presence of a borrowing constraint, a Ponzi-Scheme is excluded. This explains the absence of the usual no-Ponzi game constraint ($\lim_{z \rightarrow +\infty} e^{-\int_t^z [r(v)1 - \tau_k(v) + \frac{1}{\lambda_i}] dv} W^i(s, z) = 0$), which would be redundant.
- 12 By assumption, the unintended bequests by the rich do not go to the poor. This assumption is consistent with the findings by Gist and Figueiredo (2006). Inheritance size is correlated with net worth.
- 13 See section 4 for proofs and discussion.
- 14 See Blanchard (1985).
- 15 The labor supply choice being endogenous, it implies that efficiency per unit of labor is deduced from: $E^i(s, t) = a_i e^{-\alpha_i(t-s)} E^i(t) N^i(t) / N^i(s, t)$
- 16 The propensity to consume is independent of the date of birth when preferences are additively separable. With a CRRA:

$$\varphi^i(t) = 1 \left(\int_t^{+\infty} e^{-\int_t^v [(1-1/\sigma_{c,i})\tilde{r}(v) + \frac{1}{\lambda_i} + \delta / \sigma_{c,i}] dv} dz \right)$$

where $\sigma_{c,i}$ is the elasticity of the marginal utility of consumption.

- 17 See Calvo and Obstfeld [1988] for more details. This is only a necessary (not a sufficient) condition for time consistency. This is why I have assumed that the government is committed to its policy.
- 18 In the case of no annuitization and a binding no-borrowing constraint, the static constraint reflects that the poor consume their disposable income: $C^P(s, z) = \hat{\omega}^P(z) E^P(z-s) N^P(s, z)$ and at the optimum: $\hat{\omega}^P(z) E^P(z-s) = -\frac{U_{N^P(s, z)}}{U_{C^P(s, z)}}$.
- 19 I give a general expression of the allocations and tax rates on all dates, including the initial date t . I have incorporated the term $e^{-\left(\delta+\frac{1}{\lambda_i}\right)(s-t)} U_{C^i(s, t)} W^i(s, t)$ in \mathcal{U} . This term reflects the fact that agents born prior to t have accumulated wealth. The state of the economy follows a continuous time Markov process, with a unique and invariant probability density. Consequently, the allocation rules on all dates after t are time invariant. On a date t the allocation rules include terms relative to the financial wealth $W^i(s, t)$ of agents born prior to that date. After date t , the first order conditions do not include this term.
- 20 See conditions for the concavity of V in the online appendix.

- 21 In Judd (1985), capitalists do not work. Workers do not accumulate wealth or borrow. Workers and capitalists have an infinite horizon.
- 22 If the borrowing constraint faced by the poor binds at the optimum (workers-capitalist scenario), $T_{kB}^P = 0$. Therefore $T_{kB}^P > T_{kNB}^P$. A subsidy to the poor's capital income has no effect on their well-being if their borrowing constraint binds at the optimum. By contrast, subsidizing low income individual capital income increases their well-being when the borrowing constraint does not bind.
- 23 If at the optimum, the borrowing limits still bind, the conditions for zero capital income taxation are more stringent.