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TWO GENERALIZATIONS OF UNIQUE FACTORIZATION DOMAINS

POR

RAJ.KUMAR MARKANDA

UNIVERSIDAD DE LOS ANDES
FACULTAD DE CIENCIAS
DEPARTAMENTO DE MATEMATICA
MERIDA-VENEZUELA
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RAJ KUMAR MARKANDA, DEPARTAMENTO DE MATEMATICA UNIVERSIDAD DE LOS ANDES, MERIDA, VENEZUELA.

ABSTRACT

In this paper we first compare the two generalizations of UFD's due to Fletcher and Galovich. Utilizing the definition of a unique factorization ring and its structure theorem due to Fletcher we generalize some results on UFD's. Furthermore, we show that the concepts of an irreducible element and a prime element in a UFR are equivalent. This helps us in generalizing some results of Kaplansky. We also show that if R is a special PIR then R[X] is not a UFR.

Two generalizations of unique factorization domains have been considered by Fletcher ([2],[3]) and Galovich[4]. Let R be a commutative ring with unity. According to Galovich, R is a unique factorization ring if every non--unit a in R, $a \neq 0$, can be written as

$$a = ur_1 r_2 \text{-----} r_m,$$

where u is a unit in R and each r_i is an irreducible element of R, in a unique way i.e. if

$$a = ur_1 r_2 \cdots r_m = v s_1 s_2 \cdots s_n$$

are two irreducible decompositions of a then $m = n$ and after renumbering $s_1, \cdots, s_m$, r_i and s_i are associate. Then Galovich has proved the following

STRUCTURE THEOREM. Let R be a unique factorization ring with non-trivial zero divisors. Then either R is a special principal ideal ring or R is a local ring such that $rs = 0$ for all irreducible elements r and s of R.

We see from the above structure theorem that Galovich's definition of a unique factorization ring is very restrictive.

Now we proceed to study Fletcher's generalization of a unique factorization domain which gives a large class of such rings. Throughout, by a ring R we will mean a commutative ring with 1.

DEFINITION 1. A refinement of a factorization $a=a_1a_2 \cdots a_n$ is obtained by factoring one or more of the a_i's.

DEFINITION 2. A non-unit element r of a ring R is said to be irreducible if each factorization of r has a refinement containing r.

NOTE: According to this definition the zero of an integral domain is an irreducible element.

<u>DEFINITION 3</u>. An element p of a ring R is said to be prime if p divides ab in R implies that either p divides a or p divides b in R.

<u>DEFINITION 4</u>. Two elements a and b in R are said to be associate if a divides b and b divides a.

<u>DEFINITION 5</u>. For c in R, the set

$$U(c) = \{a \in R: abc = c \text{ for some } b \in R\}$$

is called the U-class of c.

<u>EXAMPLE</u>. In $R = \frac{Z}{6Z} = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}\}$

$$U(\bar{2}) = \{\bar{1}, \bar{5}, \bar{2}, \bar{4}\}, \quad \bar{2} = \bar{2}.\bar{4}, \ \bar{4} = \bar{2}.\bar{2}$$

Thus $\bar{2}$ and $\bar{4}$ are associate and $\bar{2}$ is irreducible in Fletcher's sense but is not irreducible in Galovich's sense.

If R is a domain then U(c) is the set of all units of R.

<u>DEFINITION 6</u>. A ring R is called a pseudo-domain if U(c) is the set of units of R for all $c \in R$, $c \neq 0$, e.g. $R = \frac{Z}{p^n Z}$ is a pseudo-domain.

The following is an immediate consequence of the definition 5.

PROPOSITION 1.

i) $a, b \in U(c)$ implies $ab \in U(c)$

ii) $a \in U(bc)$, $b \in U(c)$ implies $a \in U(c)$

iii) $U(a) \subseteq U(ab)$

iv) $U(ab) = U(a)$ for a non-zero divisor b.

DEFINITION 7. A U-decomposition of an element a in R is a factorization of a

$$a = (r_1' \, r_2' --- r_m') \, (r_1 \, r_2 --- r_n)$$

such that

i) r_i', r_j are irreducible elements of R,

$$1 \leq i \leq m, \quad 1 \leq j \leq n$$

ii) $r_i' \in U(r_1 \, r_2 --- r_n)$, $\quad 1 \leq i \leq m$

iii) $r_j \notin U(r_1 --- r_{j-1} \, r_{j+1} --- r_n)$, $\quad 1 \leq j \leq n$

EXAMPLE. In $R = \dfrac{Z}{6Z}$, $\overline{4} = \overline{2}.\overline{2}$ with $r_1' = \overline{2}$ and $r_1 = \overline{2}$.

PROPOSITION 2. If $a \in R$ has an irreducible decomposition then r has a U-decomposition.

PROOF. (see Prop 2, [2].)

DEFINITION 8. Two U-decompositions of $a \in R$

$$a = (r_1' ---- r_m')\ (r_1\ r_2 --- r_n)$$

$$= (s_1' ---- s_k')\ (s_1 ---- s_\ell)$$

are said to asociate if

i) $n = \ell$

ii) after a suitable change of numbering of the factors of the second U-decomposition, the elements r_i and s_i are associates, $1 \leq i \leq n$.

DEFINITION 9. A ring R is called a UFR if

i) every non-unit element (including 0) of R has a U-decomposition.

ii) any two U-decompositions of a non-unit element of R are associate.

The U-decomposition of the 0 element of R plays an important role in determining the structure of a UFR(see Theorem 19, [3]).

The following proposition gives the structure of a UFR which is also a pseudo-domain.

PROPOSITION 3. (Prop. 17, [3]) If R is a UFR which is also a pseudo-domain then either R is a UFD or a special PIR.

Now we study the structure of direct sum of UFR's.

PROPOSITION. Let R_1 and R_2 be two UFR's. Then the irreducible elements of $R = R_1 \oplus R_2$ are of the following types or their associate:

1) $(r_1,1)$ with r_1 irreducible element in R_1

2) $(1,r_2)$ with r_2 irreducible element of R_2 .

In particular, if R_1(respectively R_2) is a field then $(0,1)$ (respectively $(1,0)$) is an irreducible element of R.

PROOF. Clearly $(r_1,1)$ is an irreducible element of R if r_1 in R_1 is irreducible.

Let $(a,b) \varepsilon R = R_1 \oplus R_2$ be a non-unit in R. Then one of a or b is not a unit. Let a be not a unit. If $(a,b) = (a,1)\ (1,b)$ is an irreducible element of then since $(a,1) \notin u((1,b))$ and $(a,1)$ is not a unit, we should have $(1,b) \varepsilon U((a,1))$ ie. b is a unit. Now a has to irreducible, for otherwise

$$a = a_1\ a_2 \quad \text{with} \quad a_1,\ a_2 \ \varepsilon\ R, \quad a_1 \notin U(a_2)$$
$$\text{and} \quad a_2 \notin U(a_1)$$

implies that

$$(a,1) = (a_1,1)(a_2,1) \quad \text{with} \quad (a_1,1) \notin U((a_2,1))$$
$$\text{and} \quad (a_2,1) \notin U((a_1,1))$$

and thus (a,1) is not irreducible.

The case when b is not a unit is analogous.

In case R_1 is a field or an integral domain then 0 is an irreducible of R_1 and thus (0,1) is an irreducible element of $R = R_1 \oplus R_2$.

Now we give an easy and direct proof of Theorem 5, of [2], considering some additional cases not given by Fletcher.

THEOREM 1. Let R_1 and R_2 be UFR's. Then $R = R_1 \oplus R_2$ is also a UFR.

PROOF. Case i) R_1 and R_2 are both fields. Then, by Prop 4, (1,0) and (0,1) are precisely the irreducible elements of R. If a is a non-unit of R then either a =(0,0) or a =(u,0) with u a of unit of R_1 or a =(0,v) with v a unit of R_2. Then

$$a = (0,0) = (1,0)\ (0,1)$$

$$a = (u,0) = (u,1)\ (1,0)$$

$$a = (0,v) = (0,1)\ (1,v)$$

are the U-decompositions of a unique upto asociates.

Case ii) R_1 is a field and R_2 is not a field.

Let a be a non-unit of R. Then

$$a = (u,a_2) = (u,1)\ (1,a_2)$$

with u a unit of R_1 and a_2 a non-unit of R_2

$$\text{or} \quad a = (0,a_2) = (0,1)\ (1,a_2)\ .$$

Let $a_2 = (r_1'---r_m')\ (r_1---r_n)$ be a U-decomposition of a_2 in R_2. Then

$$\begin{aligned} a &= (u,a_2) \\ &= (u,1)(1,a_2) \\ &= ((u,1)(1,r_1')(1,r_2')---(1,r_m'))\ ((1,r_1)(1,r_2)---(1,r_n)) \end{aligned}$$

$$\begin{aligned} \text{or} \quad a &= (0,a_2) \\ &= (0,1)\ (1,a_2) \\ &= ((1,r_1')---(1,r_m'))((0,1)(1,r_1)---(1,r_n) \end{aligned}$$

are U-decompositions of a.

Conversely, any U-decomposition of a gives us a U-decomposition of a_2 and using the fact that R_2 is a UFP we find that any two decompositions of a are associate.

Case iii) R_1 and R_2 are not fields.

Let $(a_1,a_2) \in R_1 \oplus R_2$ be a non-unit. We consider the case when both as and a_2 are non-units. The other possibilities are analogous.

Let $\qquad a_1 = (r_1'\ r_2'----r_m')\ (r_1---r_n)$

and $a_2 = (s_1'\text{----}s_k')\ (s_1\text{----}s_\ell)$

be their U-decompositions in R_1 and R_2 respectively. Then

$$(a_1,a_2) = (a_1,1)(1,a_2)$$

$$=((r_1',1)\text{---}(r_m',1))((r_1 1)\text{----}(r_n,1))((1,s_1')\text{---}(1,s_k'))$$

$$((1,s_1)\ (1,s_2)\text{----}(1,s_\ell))$$

$$=((r_1',1)(r_2',1)\text{---}(r_m',1)(1,s_1')\text{---}(1,s_k'))$$

$$((r_1,1)(r_2,1)\text{---}(r_n,1)(1,s_1)\text{---}(1,s_\ell))$$

with

$$(r_i',1)\ \varepsilon\ U((r_1 r_2\text{---}r_n,\ s_1\text{---}s_\ell))$$

$$(1,s_j')\ \varepsilon\ U((r_1 r_2\text{---}r_n,\ s_1 s_2\text{---}s_\ell))$$

$$(r_i,1)\ \notin\ U((r_1\text{---}r_{i-1}r_{i+1}\text{---}r_n,\ s_1\text{---}s_\ell))$$

$$(1,s_j)\ \notin\ U((r_1\text{---}r_n,\ s_1\text{---}s_{j-1}\ s_{j+1}\text{---}s_\ell))$$

is a U-decomposition of (a_1,a_2).

We here use the fact that $(a,b)\ \varepsilon\ U((c,d))$ if and only if $a\ \varepsilon\ U(c)$ and $b\ \varepsilon\ U(d)$.

If (a_1,a_2) has another U-decomposition in $R_1 \oplus R_2$ then this will give us U-decompositions of a_1 and a_2 in R_1 and R_2 respectively. Comparing these U-decompositions of a_1 and a_2 with the original U-decompositions of a_1 and a_2 we find that the two U-decompositions of (a_1, a_2) are associate.

One can easily prove the following.

THEOREM 2. If $R = R_1 \oplus R_2$ is a UFR then R_1 and R_2 are UFR's.

PROPOSITION 5. If R is a special PIR then R is a UFR which is also a pseudo-domain.

PROOF. Let P = pR be the prime ideal of R such that $p^m R=0$ and $p^{m-1}R \neq 0$. Then all elements of R can be unique written in the form $u.p^n$ with u is a unit of R and $0 \leq n \leq m-1$. Now $U(up^n)$ is the set of units of R.

Using the fact that o has a U-decomposition, Fletcher proves the following.

THEOREM 3. (Theorem 19, [3]) Every UFR is a finite direct sum of UFD's and of special PIR's.

THEOREM 4. Let R be a UFR. Then an element r of R is irreducible iff r is a prime element of R.

PROOF. Comparing definitions 2 and 3 we find that a prime element is an irreducible element.

Now, let r be an irreducible element of R. By Theorem

2, R is a UFR implies that

$$R = R_1 \oplus --- \oplus R_m$$

where each R_i is either a UFD or a special PIR. Since r is irreducible, by Prop. 4, r is of the form

$$r = (1,1,---,1,r_i,1,---,1)$$

with r_i irreducible in R_i ($r_i = 0$ if R_i is a field). Since R_i is either a UFD or a special PIR, r_i irreducible in R_i implies that r_i is a prime and thus r is a prime element of R.

In view of Theorem 4, we may consider a prime decomposition of a non-unit a in R in place of an irreducible decomposition and prove the following generalizations of Theorem 3 and 5 of [5] if we don't distinguish between prime and irreducible elements.

THEOREM 5. If an element a of a ring R is a product of primes then a has a U-decompositon and any two U-decompositions are associate.

PROOF. Let $a = p_1 --- p_\ell$ be a product of primes $p_i, 1 \leq i \leq \ell$. Since each prime p_i is also an irreducible element, by Prop 2, a has a prime U-decomposition of the form

$$a = (p_1' ---- p_m')\ (p_1 --- p_n)$$

such that

$$p_i' \in U(p_1 --- p_n), \qquad 1 \le i \le m$$

$$p_j \notin U(p_1 --- p_{j-1}\ p_{j+1} --- p_n), \qquad 1 \le J \le n\ .$$

Let $\quad a = (q_1' ---- q_k')\ (q_1 --- q_\ell)$

with $\quad q_i' \in U(q_1 --- q_\ell)\ , \qquad 1 \le i \le k$

and $\quad q_j \notin U(q_1 --- q_{j-1}\ q_{j+1} --- q_\ell), \quad 1 \le j \le \ell$

be another prime U-decomposition of a.

Then

$$(p_1' --- p_n')\ (p_1 --- p_n) = (q_1' --- q_k')\ (q_1 --- q_\ell)$$

implies that p_j divides one of the primes

$$q_1', ---, q_k'\ ,\ q_1, ----, q_\ell\ .$$

Since

$$q_i' \in U(q_1 --- q_\ell), \quad \text{for} \quad 1 \le i \le k,$$

we find that $p_j/q_{j_\circ}$ for some $j_\circ$. Conversely $q_{j_\circ}/p_{j'}$ for some j'. Since $p_j \notin U(p_{j'})$ if $j \neq j'$, we find that p_j and $q_{j_\circ}$ are associate. Proceeding in this way we find that any two prime U-decompositions of a are associabe

<u>THEOREM 6</u>. A ring R is a UFR if and only if every prime

ideal of R contains some prime element p of R.

PROOF. Let R be a UFR and P be a prime ideal of R (we allow P to be the zero ideal also if R is a UFD) then $0 \varepsilon R$ has a prime U-decomposition

$$0 = (p_1'----p_m')\ (p_1---p_n)$$

implies that p_i' or $p_j \varepsilon P$ for some i or some j.

Conversely, let every prime ideal of R contain a principal prime. Let S be the set of all finite product of prime. We show that S contains all the non-units of R. Suppose that a is a non-unit of R such that $a \notin S$ (a can be o also). Since S is a saturated multiplicative subset of R, we find that $aR \cap S = \phi$. Then there exists a prime ideal P of R such that $aR \subset P$ and $P \cap S = \phi$. By hipothesis there is a prime $p \varepsilon P$. Then $P \cap S = \phi$ implies that $p \notin S$ contradicting the definition of S.

In order to study the rings of fractions of a UFR we need the following.

THEOREM 7. Let $R = R_1 + R_2$ and S be a multiplicative saturated subset of R. Set

$$S_1 = \{s_1 \varepsilon R_1 \quad : \quad (s_1, 1) \varepsilon S\}$$

$$S_2 = \{s_1 \varepsilon R_2 \quad : \quad (1, s_2) \varepsilon S\}$$

Then $$S^{-1}R \cong S_1^{-1} R_1 \oplus S_2^{-1} R_2$$

In case $(1,0) \in S$ then we take $S_2^{-1} R_2 = 0$. This will be the case when R_2 is a field.

<u>PROOF</u>. We first note that S is generated by irreducible elements of the following type

i) $(r_1,1)$ with r_1 irreducible in R_1

ii) $(1,r_2)$ with r_2 irreducible in R_2

iii) $(1,0)$ if R_2 is a field

iv) $(0,1)$ if R_1 is a field.

Case i) Both R_1 and R_2 are not fields.

Clearly S_1 and S_2 are multiplicative saturated subsets of R_1 and R_2 respectively. Then the map

$$f: R \longrightarrow S_1^{-1} R_1 \oplus S_2^{-1} R_2$$

given by $$f(a) = b((a_1,a_2)) = \left(\frac{a_1}{1} , \frac{a_2}{1} \right)$$

is a homomorphism of rings with the image of S a subset of units of $S_1 R_1 \oplus S_2 R_2$. Thus we get a homomorphism

$$\bar{f}: S^{-1}R \longrightarrow S_1^{-1} R_1 \oplus S_2^{-1} R_2$$

given by

$$\overline{f}\left(\frac{a}{s} \right) = \overline{f}\left(\frac{(a_1,a_2)}{(s_1,s_2)} \right) = \left(\frac{a_1}{s_1} , \frac{a_2}{s_2} \right)$$

which is onto. Now if

$$\overline{f}\left(\frac{a}{s} \right) = \left(\frac{a_1}{s_1} , \frac{a_2}{s_2} \right) = 0 \quad \text{in} \quad S_1^{-1} R_1 \oplus S_2^{-1} R_2$$

then $(t_1 a_1, t_2 a_2)=0$ with $t_1 \varepsilon S_1$ and $t_2 \varepsilon S_2$.

Thus $t = (t_1,t_2) \varepsilon S$ and $ta = 0$ and

$$S^{-1}R \cong S_1^{-1} R_1 \oplus S_2^{-1} R_2 .$$

Case ii) R_2 is a field and $(1,0) \varepsilon S$.

Then setting $S_1^{-1} R_2 = 0$, we find that the homomorphism $g\colon S^{-1}R \longrightarrow S_1^{-1} R_1$ given by

$$g\left(\frac{a}{s} \right) = g\left(\frac{(a_1,a_2)}{(s_1,s_2} \right) = \frac{a_1}{s_1}$$

is an isomorphism of $S^{-1}R$ onto $S_1^{-1} R_1$.

<u>Note</u>: 1) $(r_1,0) \varepsilon R_1 \oplus R_2$ with r_1 not a unit is never an irreducible element. For

$$(r_1,0) = (1,0)\ (r_1,1)$$

with $(1,0) \notin U((r_1,1))$ and $(r_1,1) \notin U((1,0))$.

2) $$R = \frac{Z}{6Z} = \frac{Z}{2Z} \oplus \frac{Z}{3Z}$$

$S = \{\bar{1}, \bar{2}, \bar{4}\}$

$= \{(\bar{1},\bar{1}), (0,\bar{2}), (0,\bar{1})\}$

Then $S^{-1}R \cong \frac{Z}{3Z}$.

THEOREM 8. Let R be a UFR and S be a multiplicative saturated subset of R. Then $S^{-1}R$ is a UFR.

PROOF. R is a UFR implies, by Theorem 3, that

$$R = R_1 \oplus R_2 \oplus \text{----} \oplus R_m$$

with each R_i either a UFD or a special PIR.

Then

$$S^{-1}R = S_1^{-1}R_1 \oplus \text{----} \oplus S_m^{-1} R_m$$

by theorem 5. In case some R_i is a field, we set $S_i^{-1} R_i=0$ if $(1,1,\text{---},1,0,1,\text{---},1) \in S$. It is well known that if R_i is a UFD then $S_i^{-1} R_i$ is a UFD and in case R_i is a special PIR then $S_i^{-1} R_i = R_i$ is again a special PIR. Then, by Theorem 2, $S^{-1}R$ is a UFD.

THEOREM 9. Let R be a UFR such that $R = R_1 \oplus \text{----} \oplus R_n$ with each R_i a UFD. Then the polinomial ring $R[x]$ is a UFR.

PROOF. $R[x] = R_1[x] \oplus \text{----} \oplus R_n[x]$ and each $R_i[x]$ a UFD implies that $R[x]$ is a UFR.

THEOREM 10. Let R be a special PIR. Then $R[x]$ is not a UFR.

PROOF. Let $P = pR$ be the unique prime ideal of R such that $P^t = 0$ and $P^{t-1} \neq 0$. We assert that x is an irreducible element of $R[x]$. For, suppose that

$$x=(a_0+ a_1x +----+ a_mx^m)(b_0+ b_1 x +---+ b_n x^n)$$

Then $$a_0b_0= 0$$

and $$a_1b_0+ a_0b_1 = 1$$

From these equations we see that a_0 and b_0 both cannot be zero.

Case i) Let $a_0\neq 0$ and $b_0\neq 0$. Then $a_0b_0= 0$ implies that $a_0= up^r$, $1 \leq r \leq t-1$ and $b_0= up^s$, $1\leq s \leq t-1$ with u and v units of R. Then

$$1 = a_0b_1 + a_1 b_0$$
$$= p(up^{r-1} b_1 + a_1 up^{s-1}),$$

a contradiction.

Case ii) Suppose that $a_0= 0$ and $b_0\neq 0$.

Then

$$x = (a_1x + a_2 x^2+---+ a_m x^m) (b_0 + b_1 x +----+ b_n x^n)$$

$=x(a_1 + a_2 x + \cdots + a_m x^{m-1}) (b_o + b_1 x + \cdots + b_n x^n)$

$= x f(x) g(x)$.

Thus $x(1-f(x)g(x)) = 0$.

If $1-f(x)g(x) \neq 0$ then x is a zero divisor and, by exercise 2, page 10 [1] , there exists $a \varepsilon R$ such that $ax = 0$, a contradiction. Thus $1-f(x)g(x) = 0$ ie $1=f(x)g(x)$. It follows that if $x = x.f(x)g(x)$ then f(x) and g(x) are units and thus x is irreducible.

Case iii) $a_o \neq 0$ and $b_o = 0$.

This is similar to case ii.

We have shown that x is an irreducible element. On the other hand x can not be a prime element of $R[x]$ as $\frac{R[x]}{(x)} \cong R$ is not a domain. Thus, by Theorem 4, $R[x]$ is not a UFR.

<u>COROLLARY</u>. If R is a UFR such that $R = R_1 \oplus \cdots \oplus R_n$ and some R_i is a special PIR then $R[x]$ is not a UFR.

<u>THEOREM 11</u>. Let R be a ring and S be a multiplicative saturated subset of R generated by regular element. Then

$$SR = R_1' \oplus R_2'$$

implies that there exist suitable S_1, S_2, R_1 and R_2 such that $R_i' = S_i^{-1} R_i$, $1 \leq i \leq 2$, and

$$R = R_1 \oplus R_2 .$$

<u>PROOF</u>. Let $r \longrightarrow \frac{r}{1} = (r_1, r_2)$ be the injection of R in $S^{-1}R = R_1' \oplus R_2'$. If $s = (s_1, s_2) \varepsilon S$ then $\frac{1}{s} = (\frac{1}{s_1}, \frac{1}{s_2})$. Thus elements of $S^{-1}R$ of the form $\frac{r}{s} = (\frac{r_1}{s_1}, \frac{r_2}{s_2})$. Taking

$$R_1 = \{r_1 : (r_1, r_2) = r \ \varepsilon \ R \ \text{ for some } r_2 \ \varepsilon \ R_2'\}$$

$$R_2 = \{r_2 : (r_1, r_2) = r \ \varepsilon \ R \ \text{ for some } r_1 \ \varepsilon \ R_1'\}$$

$$S_1 = \{s_2 : (s_1, 1) \ \varepsilon \ S\}$$

$$S_2 = \{s_2 : (1, s_2) \ \varepsilon \ S\}$$

we get the required result.

<u>THEOREM 12</u>. Let R be a ring satisfying ascending chain condition on principal ideals. Let S be the multiplicative subset of R generated by a set of irreducible regular elements. Then if $S^{-1}R$ is a UFR such that

$$S^{-1}R = R_1' \oplus \text{----} \oplus R_m'$$

with each R_i' a UFD then R is a UFR.

<u>PROOF</u>. By Theorem 11, $S^{-1}R = R_1' \oplus \text{----} \oplus R_m'$

$$= S_1^{-1} R_1 \oplus \text{---} \oplus S_m^{-1} R_m$$

with $R = R_1 \oplus \cdots \oplus R_m$, and each S_i is a multiplicative saturated subset of R_i generated by irreducible regular elements. Also each R_i has an ascending chain condition on principal ideals. Then, by Theorem 177 [5], $R_i' = S_i^{-1} R_i$ is a UFD implies R_i is a UFD for $1 \leq i \leq m$. It now follows that R is a UFR.

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