Energy-Momentum Tensor Valued Distributions for the Schwarzschild and Reissner-Nordstrom Geometries

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An approach to computing, withing the framework of distribution theory, the distributional valued energy-momentum tensor for the Schwarzschild spacetime is disscused. This approach avoids the problems associated with the regularization of singularities in the curvature tensors and shares common features with the by now standard treatment of discontinuities in General Relativity. Finally, the Reissner-Nordstrom spacetime is also considered using the same approach.

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I. INTRODUCTION

The energy-momentum tensor **T** associated to a point particle has an unambiguos character either if the particle is considered in a flat minkowskian spacetime or as a test particle in a curve background geometry. Obtained by a constructive procedure, invoking the minimal coupling principle or defined as the variational derivative of the point particle action with respect to the metric, the result in an obvious notation, is

$$T^{ab} = m\frac{\delta(\vec{x} - \vec{x}_p)}{\sqrt{-g}} u^a \frac{dx^b}{dt} \tag{1}$$

If the particle is at rest with respect to the chosen reference frame, the only surviving component is the $t{-}t$ component, i.e., the one representing energy density, as it should be for a particle with zero momentum.

The situation changes radically in general relativity if the point particle is acting as the gravitational field source. The singular behaviour of the metric and related geometrical quantities at the particle location turns the mathematical meaning of the energy tensor rather imprecise. Nevertheless, on physical grounds or suggested by application of the variational principle leading to Einstein field equations, it seems plausible that the only non vanishing component of the energy-momentum tensor for a static point source is again the $t - t$ component, with support at the particle position, and that is what it should be expected, whatever techniques are employed to find **T**.

In the classical theory of gravitation one is led to consider the Einstein field equations which are, in general, quasilinear partial differential equations involving second order derivatives for the metric tensor. Hence, continuity of the first fundamental form is expected and at most, discontinuities in the second fundamental form, the coordinate independent statements appropriate to consider 3-surfaces of discontinuity in the spacetime manifolfd of General Relativity [1].

The use of classical distribution theory, in order to incorporate singular information into General Relativity, was advocated some time ago [2,3]. However, it has been shown that the singular parts of the Riemann tensor must be of support on a submanifold of dimension $d \geq 3$, in order to have well defined tensor distributions for it and its contractions [4]. In spite of this, several attemps have been made to regularize the Schwarzschild [5,6] and Kerr-Newman spacetimes [7,8] to obtain, after suitable limiting processes, the distributional energy-momentum tensors which play the role of point sources for these geometries with results that seem to depend on the regularization prescription [6].

Besides the fact that the notion of a point source may has no precise mathematical meaning withing the framework of the nonlinear Einstein's theory of general relativity, some failures of previous works can be, to some extend, clearly

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stablished and then circumvented. In order to reveal some of the weaknesses of the above forementioned regularization approaches, we will rederive the results of Ref. [5] following a different approach. Consider the line element

$$ds^2 = h(r)dt^2 - h(r)^{-1}dr^2 + r^2 d\theta^2 + r^2 sin^2\theta \ d\phi^2, \tag{2}$$

where

$$h(r) = -1 + \frac{2m}{r}\Theta(r-\epsilon), \tag{3}$$

with $r < 2m$, $\Theta(r-\epsilon)$ is the Heaviside function and the limit $\epsilon \to 0$ is understood. Equation (2) with h as given in (3) can be considered as a regularized version of the Schwarzschild line element in curvature coordinates. From equation (2), the calculation of the Einstein tensor proceeds in a straighforward manner. It gives

$$G^t{}_t = G^r{}_r = -\frac{1}{r}\frac{dh}{dr} - \frac{1}{r^2}(1+h), \tag{4}$$

and

$$G^\theta{}_\theta = G^\phi{}_\phi = -\frac{1}{2}\frac{d^2h}{dr^2} - \frac{1}{r}\frac{dh}{dr}. \tag{5}$$

Next, taking the derivatives in the sense of distributions one finds

$$G^t{}_t = G^r{}_r = -2m\frac{\delta(r-\epsilon)}{r^2}, \tag{6}$$

and

$$G^\theta{}_\theta = G^\phi{}_\phi = -m\frac{\delta(r-\epsilon)}{r^2} - \frac{m}{\epsilon}\frac{d}{dr}\delta(r-\epsilon) = m\frac{\delta(r-\epsilon)}{r^2} - \frac{m\epsilon}{r^2}\frac{d}{dr}\delta(r-\epsilon). \tag{7}$$

We get in the limit $\epsilon \to 0$

$$G^t{}_t = G^r{}_r = -\frac{1}{2}G^\theta{}_\theta = -\frac{1}{2}G^\phi{}_\phi = -2m\frac{\delta(r)}{r^2}, \tag{8}$$

which is exactly the result obtained in Ref. [5] using smoothed versions of the Heaviside function $\Theta(r-\epsilon)$. This should be contrasted with what is the expected result

$$G^a{}_b = -8\pi m \delta^3(\vec{x})\delta^a_0\delta^0_b. \tag{9}$$

Note that the strategy followed here to obtain equations (6,7) suggest the possibility of attain a distributional meaning to curvature tensors by taking derivatives of the metric in the sense of distributions, an idea we shall pursue later. A second approach to the problem [7], using the Kerr-Schild *ansatz* for the metric tensor, has been claimed to give the correct result (9), assuming that even under the regularization procedure the metric maintains its Kerr-Schild form. Nevertheless, it can be shown that in Kerr-Schild coordinates, without the constraint imposed by the Kerr-Schild *ansatz*, the same $G^a{}_b$ as given by (8) is obtained.

Several regularization schemes of this kind can be questioned on formal grounds. The unpleasant features are traceable, at least in part, to the fact that the metric obtained from the line element (2,3) does not satisfies reasonable continuity requirements on the 3-surface $r = \epsilon$. Furthermore, as is well known for the Schwarzschild solution in curvature coordinates, r changes in character from a spacelike coordinate for $r > 2m$ to a timelike one for $r < 2m$. Hence for values of $r < 2m$ it is necessary to use a nonstatic system of coordinates. The discontinuity of the metric tensor and the change of the signature cast doubt over the previous results obtained from regularizing the Schwarzschild metric in these coordinates.

In order to circumvent these problems we will consider a different approach which is closer to the classical theory of distributions view and that has a parallel into the theory of surface layers in General Relativity [1]. Assuming that the Schwarzschild geometry can be considered as being generated by an energy-momentum tensor with support in the singularity, one method of attacking the problem is to replace the point source by a uniform simple layer on a sphere of radius ρ_0. As $\rho_0 \to 0$ the configuration tends to a point but for radius $> \rho_0$ we always have an exact solution of Einstein's equations that looks exactly like the Schwarzschild solution. In a certain sense, we shall have to deal with a sequence of distributions, or with distributions depending on an arbitrary parameter ρ_0.

The paper is organized as follows: in section **II** we briefly recall the construction of surface stress energy tensors for spherical thin shells, following the treatment of junction conditions of Israel [1]. Some features of this construction are also discussed. In section **III**, using distributional techniques, the energy-momentum tensor on the whole manifold for a spherical thin shell is derived and showed to be in complete agreement with the results of previous section. Section **IV** is devoted to the point particle limit of the spherical shell, viewed as the limit of a sequence of distributions. Following the same approach, it is shown in section **V**, how these results are extended to the Reissner-Nordstrom spacetime. Finally, some concluding remarks are given in Section **VI**.

We shall use geometrized units in which $c = G = 1$, and the signature of the metric is $(-, +, +, +)$

II. THE SURFACE LAYER FORMALISM.

This section will be intented to apply the formalism of singular hypersurfaces in order to construct the surface stress tensor of a spherical thin shell. We shall follow Lake notation [9], with minor differences in conventions.

Let us consider a spherical thin shell Σ, a singular time-like hypersurface which divides the spacetime in two regions: the interior region V^-, described by flat minkowskian geometry and V^+, the exterior spacetime, chosen as a spherically symmetric solution to Einstein field equations. Both regions $V^\pm$ will be decribed by a single set of isotropic coordinates, $x^a = (t, \rho, \theta, \phi)$ with respect to which the line element is

$$ds^2 = -A^2 dt^2 + B^2(d\rho^2 + \rho^2 d\theta^2 + \rho^2 \sin^2\theta \; d\phi^2) \tag{10}$$

where for the exterior geometry $A = A(\rho)$ and $B = B(\rho)$, are known functions of the radial isotropic coordinate and for the interior region, $A = A(\rho_0) \equiv A_0$ and $B = B(\rho_0) \equiv B_0$ are constants. The surface layer is represented in the chosen frame by

$$f(\rho) = \rho - \rho_0 = 0 \tag{11}$$

therefore, the components of the metric tensor matches naturally and smoothly across the shell, corresponding to the well defined character of the intrinsic geometry of Σ, in fact, the induced metric on Σ takes the form

$$ds^2_\Sigma = -A_0^2 dt^2 + B_0^2 \rho_0^2 (d\theta^2 + \sin^2\theta \; d\phi^2) \tag{12}$$

where intrinsic coordinates $\xi^i = (t, \theta, \phi)$ are used and the 3-metric elements induced are

$$g_{ij} = g_{ab} \frac{\partial x^a}{\partial \xi^i} \frac{\partial x^b}{\partial \xi^j} \tag{13}$$

For future purpose we note here that the relation between coordinates of $V^\pm$ and intrinsic coordinates on Σ are

$$\frac{\partial x^a}{\partial \xi^i} = \delta^a_i \tag{14}$$

On the other hand, the extrinsec geometry of Σ is not well defined, i.e., the extrinsec curvature tensor, $\mathbf{K} \equiv -\frac{1}{2}\mathcal{L}_\mathbf{n}\mathbf{g}$ induced on the layer is diferent as evaluated from its embedding in V^+ or V^- and the surface stress energy tensor is related to the discontinuity of $\mathbf{K}$ through the Lanczos equation

$$8\pi S^i{}_j = [K^i{}_j] - \delta^i_j \mathrm{Tr}\,[\mathbf{K}] \qquad i, j = (t, \theta, \phi) \tag{15}$$

$$S^\rho{}_\rho = S^\rho{}_j = 0$$

where square brakets denotes a discontinuity across de layer as evaluated in V^+ and V^-, i.e., $[f] \equiv f^+ - f^-$. Our task is to perform the calculations to obtain the surface stress energy tensor of the layer. From its definition it is easy to see that

$$K_{ij} = -\nabla_b n_a \frac{\partial x^a}{\partial \xi^i} \frac{\partial x^b}{\partial \xi^j} \tag{16}$$

n_a being the components of $\mathbf{n}$, the outward unit normal field to the 3-surface Σ. Using (11) and (10) the normal is found to be

$$n_a = B\delta^{\rho}{}_a \tag{17}$$

With the usual formula for the covariant derivative and equations (14) and (17), equation (16) yields the simple relation

$$K_{ij} = \; B\,\Gamma^{\rho}{}_{ij} \;, \tag{18}$$

where Γ stand for the usual Christoffel symbols. Thus, raising indexes, and performing the calculations for V^+ and for V^- we obtain

$$[K^t_t] = -\frac{A'}{AB}, \qquad\qquad [K^{\theta}_{\theta}] = [K^{\phi}{}_{\phi}] = -\frac{B'}{B^2} \tag{19}$$

where and hereafter primes denotes differentiation with respect to ρ and it is understood that quantities must be evaluated at Σ after performing the derivations. Finally, using equation (15) for the surface energy tensor, we arrive to the simple results

$$S^t{}_t = \frac{1}{4\pi B}\frac{B'}{B} \tag{20}$$

and

$$S^{\theta}_{\theta} = S^{\phi}{}_{\phi} = \frac{1}{8\pi B}\frac{(AB)'}{AB} \tag{21}$$

Let us apply the above results to obtain the surface stress tensor for a spherical layer of total gravitational mass m. Therefore A and B will be the metric elements corresponding to Schwarzschild vacumm solution in isotropic coordinates

$$A_S = (1 - \frac{m}{2\rho})(1 + \frac{m}{2\rho})^{-1} \qquad \text{in} \quad V^+ \tag{22}$$

and

$$B_S = (1 + \frac{m}{2\rho})^2 \qquad \text{in} \quad V^+ \tag{23}$$

where as usual, m is the gravitational mass as measured at infinite. From (20) and (21), **S** can be readily obtained as

$$S^t{}_t = \; -\frac{m}{4\pi\rho_0^2}(1 + \frac{m}{2\rho_0})^{-3} \tag{24}$$

and

$$S^{\theta}_{\theta} = S^{\phi}{}_{\phi} = \frac{m^2}{16\pi\rho_0^3}\;(1 + \frac{m}{2\rho_0})^{-3}(1 - \frac{m}{2\rho_0})^{-1} \tag{25}$$

The surface energy density of the layer σ, defined as $\sigma \equiv S_{ij}u^iu^j$, where u^i are the components of the intrinsic tangent vector, $u^i = A^{-1}\delta^i_t$, is given by $\sigma = S^t_t$. The energy of the shell can be evaluated by integrating σ over the proper area of the 2-sphere t= *const.* and ρ = *const.* A simple calculation gives

$$\mathcal{E} = \int S_{ij}u^iu^j \;\; da = \int \sigma\, da = m + \frac{m^2}{2\rho_0}. \tag{26}$$

III. THE DISTRIBUTIONAL APPROACH TO THIN SHELLS.

Consider again the line element

$$ds^2 = -A^2(\rho)dt^2 + B^2(\rho)(d\rho^2 + \rho^2 d\theta^2 + \rho^2 sin\theta \; d\phi^2), \tag{27}$$

$$A(\rho) = \begin{cases} \dfrac{(1-\frac{m}{2\rho_0})}{(1+\frac{m}{2\rho_0})} & (\rho < \rho_0) \\ \dfrac{(1-\frac{m}{2\rho})}{(1+\frac{m}{2\rho})} & (\rho > \rho_0) \end{cases}, \tag{28}$$

$$B(\rho) = \begin{cases} (1+\frac{m}{2\rho_0})^2 & (\rho < \rho_0) \\ (1+\frac{m}{2\rho})^2 & (\rho > \rho_0) \end{cases}, \tag{29}$$

which corresponds to the geometry produced by a very thin spherical shell of isotropic radius ρ_0: inside the sphere, the metric is equivalent to the flat-space Minkowski metric and outside it agrees with the Schwarzschild solution written in isotropic coordinates, provided we restrict ourselves to values of ρ_0 satisfying the condition $\rho_0 > \frac{1}{2}m$.

From (27) we obtain for the Einstein tensor

$$G^t{}_t = \frac{2B''B\rho - B'^2\rho + 4B'B}{B^4\rho}, \tag{30}$$

$$G^\rho{}_\rho = \frac{B'^2A\,\rho + 2B'AB + 2A'B'B\,\rho + 2A'B^2}{AB^4\rho}, \tag{31}$$

$$G^\theta{}_\theta = G^\phi{}_\phi = \frac{B'AB + B''AB\rho + A'B'^2 - B'^2A\rho + A''B^2\rho}{AB^4\rho} \tag{32}$$

Because of the linear character of the theory of distributions, the differential equations satisfied by them should be linear. The above expressions can not be written in the form of linear differential operators acting on A and B nor even on some funtions of these. To what extent can the Einstein tensor be regarded as a distribution? What we shall consider here is an alternative that one should expect to find useful in many nonlinear problems. In classical analysis, A and B are locally integrable infinitely differentiable functions of variable ρ except at $\rho = \rho_0$ where A' and B' are discontinuos and $A''$and B'' fail to exist. However, differentiating A and B in the sense of distributions, A' and B' are now distributions with jump discontinuities at Σ with resulting $f\delta(\rho - \rho_0)$ terms in the expressions for A'' and B'', with f the corresponding jump of the first derivative. From equations (30-32), this procedure provides us with a physically sensible well defined distributional tensor G^a_b given by

$$G^t{}_t = -\frac{1}{\rho^2AB^3}2m(1-\frac{m}{2\rho_0})\delta(\rho-\rho_0), \tag{33}$$

$$G^\rho{}_\rho = 0, \tag{34}$$

$$G^\theta{}_\theta = G^\phi{}_\phi = \frac{1}{2\rho^2AB^3}\frac{m^2}{\rho_0}\delta(\rho-\rho_0), \tag{35}$$

with support on the 3-surface Σ. This confirm the interpretation of equations (27-29) as the geometry whose source corresponds to a uniform surface layer density spread on the 3-surface Σ, i.e., to an energy-momentum tensor $\mathbf{T}$ with support on Σ:

$$T^t{}_t = -\frac{1}{8\pi\rho^2AB^3}2m(1-\frac{m}{2\rho_0})\delta(\rho-\rho_0), \tag{36}$$

$$T^\rho{}_\rho = 0, \tag{37}$$

$$T^\theta{}_\theta = T^\phi{}_\phi = \frac{1}{16\pi\rho^2AB^3}\frac{m^2}{\rho_0}\delta(\rho-\rho_0), \tag{38}$$

where use have been made of the Einstein equation $\mathbf{G} = 8\pi\mathbf{T}$. We recall that, the linear character of the theory of distributions avoids a more rigorous justification of the above result. However, its correctness follows directly from the relation between the surface energy-momentum tensor $S^a{}_b$ on the 3-surface Σ, equations (24,25), and the energy-momentum tensor T^a_b on the whole manifold (36-38), namely [12]

$$S^a{}_b \equiv \lim_{\epsilon\to 0}\left[\int_{-\epsilon}^{+\epsilon} T^a{}_b dn\right], \tag{39}$$

with $dn = Bd\rho$ being the proper distance element measured perpendicularly through the 3-surface Σ.

The energy-momentum tensor $\mathbf{T}$ can be cast in the form of the energy-momentum tensor of an anisotropic fluid

$$T^a_{\ b} = \zeta u^a u_b + P_\perp h^a_{\ b} \tag{40}$$

where ζ is the proper energy density, u^a are the components of the unit timelike four velocity, $P_\perp$ is the tangential pressure and $h^a_{\ b}$ is the projection tensor onto the subspace orthogonal to $\mathbf{u}$ and $\mathbf{n}$,

$$h^a_{\ b} = \delta^a_b + u^a u_b - n^a n_b \tag{41}$$

with

$$\zeta = \frac{m}{4\pi\rho^2 AB^3}(1 - \frac{m}{2\rho_0})\delta(\rho - \rho_0) \tag{42}$$

and

$$P_\perp = \frac{m^2}{16\pi\rho^3 AB^3}\delta(\rho - \rho_0). \tag{43}$$

Note that the calculation of the energy of the layer as measured by an observer with velocity $\mathbf{u}$ gives

$$\mathcal{E} \equiv \int T_{ab}\, u^a u^b d^3\sigma = \int \zeta\, d^3\sigma = m(1 + \frac{m}{2\rho_0}), \tag{44}$$

where $d^3\sigma$ is the volumen element of the rest space of the observer, in complete agreement with (26). Furthermore, the Tolman formula [10,11] for the total energy of a static and asymptotically flat spacetime

$$\mathcal{E}_T = \int (T^\rho_{\ \rho} + T^\theta_{\ \theta} + T^\phi_{\ \phi} - T^t_{\ t})\sqrt{-g}d^3x, \tag{45}$$

with g the determinant of the four dimensional metric and d^3x the coordinate volume element, gives

$$\mathcal{E}_T = m, \tag{46}$$

as it should be. Interesting enough, note that the difference between the total energy as measured in infinite and the energy of the shell, is $-m^2/2\rho_0$, which is the expression for the gravitational newtonian energy of a thin shell; recall however that the radial coordinate ρ_0 is not the proper radius of the shell.

IV. THE POINT PARTICLE LIMIT AND THE SCHWARZSCHILD GEOMETRY.

Now we consider the limit in which the spherical layer tends to a point particle and let us comment on the limiting procedure. It should be noted that the value of $m/2\rho_0$ must be keep fixed and satisfying the condition $m/2\rho_0 < 1$ in order to have a well defined metric tensor, equations (27,28,29) for all ρ. Furthermore, the static character of this spacetime reduce the distributional evaluation of $\lim_{\rho_0\to 0} G^a_{\ b}$ to a three-dimensional problem on the 3-surfaces $t = const$. For $m/2\rho_0$ fixed, it turns out that the sequence of distributions $G^a_{\ b}$ is such that $\lim_{\rho_0\to 0}\langle G^a_{\ b}, \Phi\rangle$ exist for each Φ. Let us ilustrate the evaluation of $\lim_{\rho_0\to 0} G^a_{\ b}$ with the explicit calculation of $\lim_{\rho_0\to 0} G^t_{\ t}$ where the condition $m/2\rho_0 = fixed$ is understood. We have

$$\begin{aligned}
\langle G^t_{\ t}, \Phi(\vec{x})\rangle_{\vec{x}} &= \left\langle -\frac{1}{\rho^2 AB^3} 2m(1 - \frac{m}{2\rho_0})\delta(\rho - \rho_0), \Phi(\vec{x}) \right\rangle_{\rho,\theta,\phi} \\
&= -\int_0^\infty d\rho \int_0^\pi d\theta \int_0^{2\pi} d\phi\, \rho^2 B^3 \sin\theta \;\frac{1}{\rho^2 AB^3} 2m(1 - \frac{m}{2\rho_0})\delta(\rho - \rho_0)\Phi(\rho,\theta,\phi) \\
&= -2m(1 + \frac{m}{2\rho_0}) \int_0^\pi d\theta \int_0^{2\pi} d\phi \sin\theta\; \Phi(\rho_0,\theta,\phi).
\end{aligned}$$

Taking into account that

$$\lim_{\rho_o\to 0} m(1 + \frac{m}{2\rho_0}) = \lim_{\rho_o\to 0}\left(m + 2\rho_0(\frac{m}{2\rho_0})^2 \right) = m,$$

it follows that

$$\lim_{\rho_o\to 0}\left\langle {G^t}_t,\Phi(\vec{x})\right\rangle_{\vec{x}} = \lim_{\rho_o\to 0} -2m(1+\frac{m}{2\rho_0})\int_0^\pi d\theta\int_0^{2\pi} d\phi\sin\theta\ [\Phi(\vec{0})+(\Phi(\rho,\theta,\phi)-\Phi(\vec{0})]$$
$$= \lim_{\rho_o\to 0}\left[-8\pi m(1+\frac{m}{2\rho_0})\Phi(\vec{0})-2m(1+\frac{m}{2\rho_0})\int_0^\pi d\theta\int_0^{2\pi} d\phi\sin\theta\ (\Phi(\rho,\theta,\phi)-\Phi(\vec{0})\right]$$
$$= -8\pi m\Phi(\vec{0}) = \left\langle -8\pi m\delta^3(\vec{x}),\Phi(\vec{x})\right\rangle_{\vec{x}}.$$

Hence

$$\lim_{\rho_o\to 0} {G^t}_t = -8\pi m\delta^3(\vec{x}) \tag{47}$$

The evaluation of the limit for the remaining non zero components of ${G^a}_b$ proceeds in the same manner. The results are

$$\lim_{\rho_o\to 0} {G^\theta}_\theta = \lim_{\rho_o\to 0} {G^\phi}_\phi = 0 \tag{48}$$

It them follows that this limit is a well defined distribution given by

$${G^a}_b = -8\pi m\delta^3(\vec{x})\delta^a_0\delta^0_b, \tag{49}$$

which implies *via* Einstein equations,

$${T^a}_b = -m\delta^3(\vec{x})\delta^a_0\delta^0_b, \tag{50}$$

as expected.

V. THE REISSNER-NORDSTROM SPACETIME.

Next, let us consider the case in which the shell has a uniformly distributed charge e. Hence, the V^+ spacetime line element is given by the Reissner-Nordstrom exterior solution to the Einstein-Maxwell equations, appropiately written in isotropic coordinates, i.e.,

$$A_{RN} = \frac{(1-\frac{m^2}{4\rho^2})+\frac{e^2}{4\rho^2}}{(1+\frac{m}{2\rho})^2-\frac{e^2}{4\rho^2}} \qquad \text{in} \quad V^+ \tag{51}$$

and

$$B_{RN} = (1+\frac{m}{2\rho})^2-\frac{e^2}{4\rho^2} \qquad \text{in} \quad V^+ \tag{52}$$

It follows from (20) and (21) that

$${S^t}_t = -\frac{1}{8\pi\rho_0^2}\left[2m(1+\frac{m}{2\rho_0})-\frac{e^2}{\rho_0}\right]B_{RN}^{-2}(\rho_0) \tag{53}$$

and

$${S^\theta}_\theta = {S^\phi}_\phi = \frac{m^2-e^2}{16\pi\rho_0^3}\ B_{RN}^{-2}(\rho_0)\ A_{RN}^{-1}(\rho_0). \tag{54}$$

Some comments are in order. First observe that in the extreme Reissner-Nordstrom solution, $e=\pm m$, the tangential stresses vanish. This is explained if we realize that $e=\pm m$ is precisely the Bonnor condition [13] for a selfgravitating pressureless charged fluid to be in equilibrium, therefore in this case we are dealing with a dust thin shell. Secondly, note that the calculation of the energy contained in the charged layer, integrating (53) as was done in (26), gives

$$\mathcal{E} = m(1 + \frac{m}{2\rho_0}) - \frac{e^2}{2\rho_0}, \tag{55}$$

the last term in this equation representing the (negative) electrostatic contribution to the energy. Finally, in the limit $e = 0$ these equations reduce to the neutral case, as expected.

Following the same approach of section **III**, the energy-momentum tensor for the spherical thin shell with a uniformly distributed charge e can be obtained. It is given by

$$T^t_{\ t} = -\frac{1}{8\pi\rho^2 A_{RN} B^3_{RN}} \left[\frac{(1 - \frac{m^2}{4\rho_0^2}) + \frac{e^2}{4\rho_0^2}}{(1 + \frac{m}{2\rho_0})^2 - \frac{e^2}{4\rho_0^2}} \right] \left[2m\big(1 + \frac{m}{2\rho_0}\big) - \frac{e^2}{\rho_0} \right] \delta(\rho - \rho_0)$$
$$- \frac{e^2}{8\pi\rho^4 B^4_{RN}} \Theta(\rho - \rho_0), \tag{56}$$

$$T^\rho_{\ \rho} = -\frac{e^2}{8\pi\rho^4 B^4_{RN}} \Theta(\rho - \rho_0), \tag{57}$$

$$T^\theta_{\ \theta} = T^\phi_{\ \phi} = \frac{1}{16\pi\rho^2 A_{RN} B^3_{RN}} \frac{m^2 - e^2}{\rho_0} \delta(\rho - \rho_0)$$
$$+ \frac{e^2}{8\pi\rho^4 B^4_{RN}} \Theta(\rho - \rho_0). \tag{58}$$

From these results one can verify, using equation (39), that the energy-momentum tensor **S** on the hipersurface Σ, equations (53,54) is in fact the "delta-like singularity" at Σ [12] of the energy-momentum tensor **T** given by (56-58). Note that the non-delta-like electromagnetic part of (56-58) is traceless. Furthermore, from (45) and (56-58), it follows the remarkable result $\mathcal{E}_T = m$, i.e., $\mathcal{E}_{\mathcal{T}}$ no contains electromagnetic contributions.

The limiting case of a point particle runs over the same lines given in Section **IV**. The result is

$$T^t_{\ t} = -m\delta^3(\vec{x}) - \frac{e^2}{8\pi\rho^4 B^4_{RN}} \Theta(\rho),$$
$$T^\rho_{\ \rho} = -\frac{e^2}{8\pi\rho^4 B^4_{RN}} \Theta(\rho),$$
$$T^\theta_{\ \theta} = T^\phi_{\ \phi} = \frac{e^2}{8\pi\rho^4 B^4_{RN}} \Theta(\rho). \tag{59}$$

as expected.

VI. CONCLUSIONS AND REMARKS.

We have shown that a succesfull approach for dealing with curvature tensor valued distribution is to first impose admisible continuity conditions on the metric tensor, and then take its derivatives in the sense of classical distributions. The distributional meaning is then equivalent to the junction condition formalism. Afterwards, through appropiate limiting procedures, it is then possible to obtain well behaved distributional tensors with support on submanifolds of $d \leq 3$, as we have shown for the energy-momentum tensors associated with the Schwarzschild and Reissner-Nordstrom spacetimes. The above procedure provides us with what is expected on physical grounds. However, it should be mentioned that the use of Colombeau's new generalized functions [14] in order to obtain distributional curvatures [15,16], may renders a more rigorous setting for discussing situations like the ones considered in this paper.

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