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Non-Abelian Antisymmetric-Vector Coupling from Self-Interaction

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Abstract

A non-abelian coupling between antisymmetric fields and Yang-Mills fields proposed by Freedman and Townsend several years ago is derived using the self-interaction mechanism.

1 INTRODUCTION

Abelian second-rank antisymmetric fields [1] play an essential role in strings and supergravity theories and have been extensively studied in the last decades [2] [3] [4] [5]. In free theories they describe massless and spinless particles and appear in many contexts, for instance, arising as mediators of the interaction between open strings with charged particles [2] and in ten dimensions, coupling with the Chern-Simons 3-form to achieve an elegant unification of Yang-Mills and supergravity [6]. In particular the Cremmer-Sherk theory [3] has received considerable attention [7] [8] due to the fact that the coupling between the abelian antisymmetric field and a Maxwellian field through a topological BF term leads to massive propagations which are compatible with gauge invariances. Moreover, Allen, et. al. [7] have shown unitarity and renormalizability of the Cremmer-Sherk theory. This fact motivates the non-abelian generalization of the model and several attempts have been proposed [9]. Simultaneously, other alternatives for non-abelian massive vector bosons without the presence of Higgs field have been proposed in the last year [10].

The non-abelian extension of antisymmetric theories was achieved by Freedman and Townsend [4] starting from a first-order formulation where

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the antisymmetric field B^a_{mn} and an auxiliary vector potential are independent variables. It is worth recalling that the non-abelian generalization of the abelian S-duality theory [11] is a Freedman- Townsend theory [12]. In their work, Freedman and Townsend proposed the non-abelian generalization of the Cremmer-Sherk theory. In this letter, starting from an appropiate first-order formulation for the Cremmer-Sherk theory, we will derive the non-abelian generalization using the self-interaction mechanism [13], which has been succesfully applied to formulate Yang-Mills, gravity [13], supergravity [14]. topologically massive Yang-Mills [15] and Chapline-Manton [16] theories.

2 THE ABELIAN MODEL

Our starting point will be a first-order formulation for the Cremmer-Sherk theory. This is realized introducing an auxiliary vector field (v_m) ala Freedman-Townsend. The action is written down as [17]

$$\begin{aligned} I &= < -\frac{1}{4}\mu\epsilon^{mnpq}B_{mn}[\partial_p v_q - \partial_q v_p] - \frac{1}{2}\mu^2 v^m v_m - \frac{1}{2}\mu\epsilon^{mnpq}B_{mn}\partial_p A_q \\ &+ \frac{1}{4}F_{mn}F^{mn} - \frac{1}{2}F^{mn}[\partial_m A_n - \partial_n A_m] > \end{aligned} \quad (1)$$

where <> denotes integration in four dimensions. All the fields involved have mass dimensions and μ is a mass parameter. There are two sets of abelian gauge invariances:

$$\delta_\lambda A_m = \partial_m \lambda, \quad \delta_\lambda F_{mn} = 0 \quad (2)$$

$$\delta_\zeta B_{mn} = \partial_m \zeta_n - \partial_n \zeta_m, \quad \delta_\zeta v_m = 0. \quad (3)$$

Independent variations in v_m, B_{mn}, F_{mn} and A_m lead to the following equations of motion

$$v^m = -\frac{1}{6\mu}\epsilon^{mnpq}H_{npq}, \quad (4)$$

$$\epsilon^{mnpq}\partial_p[v_q + A_q] = 0, \quad (5)$$

$$F_{mn} = \partial_m A_n - \partial_n A_m, \quad (6)$$

$$\partial_p F^{pm} = \frac{1}{6}\mu\epsilon^{mnpq}H_{npq} \quad (7)$$

where $H_{mnp} \equiv \partial_m B_{np} + \partial_n B_{pm} + \partial_p B_{mn}$ is the field strength associated with the antisymmetric field. The Cremmer-Sherk action is obtained after substituing equations (4) and (6) in (1):

$$I_{CrSc} = -\frac{1}{4} F_{mn[A]} F^{mn}_{[A]} - \frac{1}{12} H_{mnp[B]} H^{mnp}_{[B]} - \frac{1}{4} \mu \epsilon^{mnpq} B_{mn} F_{pq[A]}. \quad (8)$$

On the other hand, equation (5) can be solved(locally) for the v field,

$$v_m = -[A_m + \frac{1}{\mu} \partial_m \phi], \quad (9)$$

where ϕ is a scalar field. Substituting this solution in the action I, the Stuckelberg formulation for massive abelian vector bosons is obtained

$$I_{St} = -\frac{1}{4} F_{mn[A]} F^{mn}_{[A]} - \frac{1}{2} \mu^2 [A_m + \frac{1}{\mu} \partial_m \phi][A^m + \frac{1}{\mu} \partial^m \phi]. \quad (10)$$

As it is well known, both formulations(Stuckelberg and Cremmer-Sherk) are equivalent descriptions of massive abelian gauge invariant vectorial theories and propagate three degrees of freedom. This equivalence is reflected by the fact that they are connected by duality [18]. Indeed, since the scalar field appears in equation (10) only through its derivative, we can apply the dualization method due to Nicolai and Townsend [19], which consist in replacing $\partial_m \phi$ by $\frac{1}{2} l_m$ and adding a new term to equation (10): $\epsilon B \partial l$, i.e.

$$I_{Stmod} = -\frac{1}{4} F_{mn[A]} F^{mn}_{[A]} - \frac{1}{2} \mu^2 [A_m + \frac{1}{2\mu} l_m][A^m + \frac{1}{2\mu} l_m] + \frac{1}{4} \epsilon^{mnpq} B_{mn} \partial_p l_q. \quad (11)$$

At this stage, B_{mn} is a Lagrange multiplier forcing the constraint $\partial_m l_n - \partial_n l_m = 0$ whose local solution is $l_m = 2\partial_m \phi$. Now, if we eliminate l_m via its equation of motion

$$l^m = \frac{1}{3} \epsilon^{mnpq} H_{npq} - 2\mu A^m \quad (12)$$

and go back to equation (11), the Cremmer-Sherk action is recovered.

Finally, let us recall that the second-order field equations can be written as

$$\partial_p F^{pm} = J^m, \quad \partial_p H^{pmn} = J^{mn}, \quad (13)$$

where

$$J^m = \frac{1}{6} \mu \epsilon^{mnpq} H_{npq} \quad \text{and} \quad J^{mn} = \frac{1}{2} \mu \epsilon^{mnpq} F_{pq} \quad (14)$$

are "topological" currents in the sense that they are conserved without using the equations of motion.

3 THE SELF-INTERACTION PROCESS

Now, we extend the first-order action, equation (1), by introducing a triplet of free abelian antisymmetric fields B^a_{mn} coupled with a triplet of free abelian vector fields A^a_m, $(a = 1, 2, 3)$

$$\begin{aligned} I_o &= < -\frac{1}{4}\mu\epsilon^{mnpq}B^a_{mn}[\partial_p v^a_q - \partial_q v^a_p] - \frac{1}{2}\mu^2 v^{am}v^a_m - \frac{1}{2}\mu\epsilon^{mnpq}B^a_{mn}\partial_p A^a_q \\ &+ \frac{1}{4}F^a_{mn}F^{amn} - \frac{1}{2}F^{amn}[\partial_m A^a_n - \partial_n A^a_m] > \end{aligned} \quad (15)$$

Besides the local gauge transformations

$$\delta_\lambda A^a_m = \partial_m \lambda^a, \quad \delta_\lambda F^a_{mn} = 0 \quad (16)$$

$$\delta_\zeta B^a_{mn} = \partial_m \zeta^a_n - \partial_n \zeta^a_m, \quad \delta_\zeta v^a_m = 0, \quad (17)$$

our action has two global invariances: one is a global $SU(2)$ rotation and the other is a a global symmetry associated with the Freedman-Townsend theory:

$$(I) \quad \delta_\omega X^a = g_1 \epsilon^{abc} X^b \omega^c \quad (18)$$

where $X^a = (A^a_m, F^a_{mn}, v^a_m, B^a_{mn})$ and

$$\begin{aligned} (II) \quad \delta_\rho B^a_{mn} &= g_2 \epsilon^{abc}[v^b_m + A^b_m]\rho^c_n - m \leftrightarrow n, \\ \delta_\rho v^a_m &= \delta_\rho A^a_m = \delta_\rho F^a_{mn} = 0, \end{aligned} \quad (19)$$

ω and ρ being global parameters. In principle the coupling constants g_1 and g_2 are different. We note that under type II transformations the action changes by a total derivative. The Noether currents associated to these invariances are given by

$$g_1^{-1} j^{am} = \epsilon^{abc} F^{bmn} A^c_n + \frac{1}{2}\mu\epsilon^{mnpq}\epsilon^{abc} B^b_{pq}[A^c_n + v^c_n] \quad (20)$$

and

$$g_2^{-1} K^{amn} = \frac{1}{2}\mu\epsilon^{mnpq}\epsilon^{abc}[A^b_p + v^b_p][A^c_q + v^c_q]. \quad (21)$$

These are conserved on-shell. In order to couple these currents to the action I_o we must add the corresponding self-interaction terms: I_1 and I_2 defined by:

$$j^{am} \equiv \frac{\delta I_1}{\delta_{A^a_m}}; \quad K^{amn} \equiv -2\frac{\delta I_2}{\delta_{B^a_{mn}}}. \quad (22)$$

These functional differential equations can easily be integrated. In fact, we find that

$$\begin{aligned} I_1 &= -g_1 < \frac{1}{2}\epsilon^{abc}F^{amn}A^b_m A^c_n + \frac{1}{4}\mu\epsilon^{mnpq}\epsilon^{abc}B^a_{mn}A^b_p A^c_q \\ &+ \frac{1}{2}\mu\epsilon^{mnpq}\epsilon^{abc}B^a_{mn}A^b_p v^c_q > \end{aligned} \tag{23}$$

and

$$\begin{aligned} I_2 &= -g_2 < \frac{1}{4}\epsilon^{mnpq}\epsilon^{abc}B^{amn}v^b_p v^c_q + \frac{1}{4}\mu\epsilon^{mnpq}\epsilon^{abc}B^a_{mn}A^b_p A^c_q \\ &+ \frac{1}{2}\mu\epsilon^{mnpq}\epsilon^{abc}B^a_{mn}A^b_p v^c_q > \end{aligned} \tag{24}$$

However, these two terms have overlapping parts. This situation is akin to what happens in the derivation of supergravity from self-interaction [14]. In order to overcome this obstacle we must require equality of the coupling constants: $g \equiv g_1 = g_2$ and write down the self-interaction action as

$$\begin{aligned} I_{SI} &\equiv -g < \frac{1}{2}\epsilon^{abc}F^{amn}A^b_m A^c_n + \frac{1}{4}\epsilon^{mnpq}\epsilon^{abc}B^a_{mn}v^b_p v^c_q \\ &+ \frac{1}{4}\mu\epsilon^{mnpq}\epsilon^{abc}B^a_{mn}A^b_p A^c_q + \frac{1}{2}\mu\epsilon^{mnpq}\epsilon^{abc}B^a_{mn}A^b_p v^c_q > \end{aligned} \tag{25}$$

Actually, we have that

$$j^{am} \equiv \frac{\delta I_{SI}}{\delta_{A^a_m}} \quad \text{and} \quad K^{amn} \equiv -2\frac{\delta I_{SI}}{\delta_{B^a_{mn}}}. \tag{26}$$

The self-interaction mechanism stops here since no other derivative terms appear in I_{SI}. Finally, the full non-abelian theory is

$$\begin{aligned} I &= I_o + I_{SI} \\ &= < -\frac{1}{4}\mu\epsilon^{mnpq}B^a_{mn}[F^a_{pq} + f^a_{pq} + 2\epsilon^{abc}A^b_p v^c_p] - \frac{1}{2}\mu^2 v^a_m v^a m - \frac{1}{4}F^a_{mn}F^{amn} >, \end{aligned} \tag{27}$$

where

$$F^a_{mn} \equiv \partial_m A^a_n - \partial_n A^a_m + g\epsilon^{abc}A^b_m A^c_n \tag{28}$$

and

$$f^a_{mn} \equiv \partial_m v^a_n - \partial_n v^a_m + g\epsilon^{abc} v^b_m v^c_n \tag{29}$$

which is just that proposed by Freedman and Townsend (equation (2.15) in their paper). As usual, the self-interaction process combines the abelian gauge transformations with the global ones giving rise to non-abelian local gauge transformations. In our case, we have

$$\begin{aligned} \delta_\alpha A^a_m &= \partial_m \alpha^a + g\epsilon^{abc} A^b_m \alpha^c & (30) \\ \delta_\alpha B^a_{mn} &= g\epsilon^{abc} B^b_{mn} \alpha^c & (31) \\ \delta_\alpha v^a_m &= g\epsilon^{abc} v^b_m \alpha^c \end{aligned}$$

and

$$\begin{aligned} \delta_\xi B^a_{mm} &= \partial_m \xi^a + g\epsilon^{abc}[A^b_m + v^b_m]\xi^c - m \leftrightarrow n & (32) \\ \delta_\xi A^a_m &= 0 = \delta_\xi v^a_m. \end{aligned}$$

The action of Freedman-Townsend, equation (27), is equivalent to massive Yang-Mills (locally) as can be shown after elimination of B^a_{mn} through its equation of motion, which said us that $A_m + v_m$ is a pure gauge.

4 CONCLUSION

In this letter, by starting with a nice abelian first-order formulation, and through the application of the self-interaction mechanism we have obtained the Freedman-Townsend theory and its corresponding gauge tranformation rules through self-interaction. The first order abelian formulation allowed us to find Cremmer-Sherk and Stuckelberg formulations for massive spin-1 theories, these later formultations are connected by duality. The BRST quantization of the massive Freedman-Townsend has been performed by Thierry-Meig [20]. Since massive Freedman-Townsend theory is equivalent (in topologically trivial manifols) to massive Yang Mills it should be interesting to attempt to connect Friedman-Townsend with others approaches dealing with massive gauge bosons without the presence of Higgs field [10].

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6 REFERENCES

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