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PERIODIC SLIDING MOTIONS 1

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PERIODIC SLIDING MOTIONS [1]

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Abstract

In this article a general characterization of the global existence of sliding motions on compact manifolds is introduced for nonlinear variable structure controlled systems. The results are applied, via several illustrative examples, to the case of periodic sliding motions in the plane.

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1. INTRODUCTION

In this article a general geometric characterization of global sliding regimes (Utkin, [1], Itkis [2]) is proposed for nonlinear Variable Structure Systems (VSS), defined in R^n, which adopt as sliding surfaces smooth manifolds bounding compact regions of R^n (henceforth such manifolds are called compact manifolds). For general background about VSS, readers are referred to the excelent tutorial by Zak, De Carlo and Matthews [3].

Under the influence of the flow map, associated to the vector field defining a nonlinear dynamical system, compact regions of the state space evolve in a rather complicated way. However, at each instant of time, the rate of change of the volume of such evolving region is very simply related, in fact it is equal, to the volume integral, over the evolving region, of the divergence of the vector field. Using this fundamental theorem, which is a stonger version of Liouville's theorem for linear systems [Arnold, 4, pp. 198 and 195], (See also Arnold [5, pp. 69, Lemma 1]), the necessary and sufficient conditions for the existence of a sliding motion on a compact manifold are translated into set inclusion conditions on the set-valued flow map generated by each possible structure of the controlled vector field. From this alternative, but particular, characterization of sliding motions, a simple necessary condition for the global existence of a sliding regime is derived. Such condition involves a difference in sign of the rate of change of the controlled volume for each available feedback structure i.e., a difference in sign of the volume integral of the divergence of the controlled vector field for each possible feedback structure. The manifold invariance condition, characteristic of smooth responses obtained from formal application of the **equivalent control** to the original controlled dynamics [1], results in having the equivalent flow map, associated to the **ideal sliding dynamics**, preserve the volume of the compact region.

Section 2 contains some background definitions and general results about sliding motions on compact manifolds. Some illustrative examples of periodic sliding modes in the plane are scattered over the section, as applications of the general results. Section 3 contains the conclusions of the article.

2. BASIC DEFINITIONS, MAIN RESULTS AND SOME APPLICATIONS

2.1 Background Results

Consider a nonlinear dynamical system, defined on R^n by

$$dx/dt = X(x,u) \qquad (2.1)$$

with u a scalar, possibly discontinuous, feedback control function and X being a smooth vector field for each given smooth u.

Definition 2.1 Let u be a given control function, the flow generated by the controlled vector field X(x,u) is the one-parameter group of transformations g^t_u of R^n such that $g^t_u : x(0) \rightarrow x(t)$ where x(t) is a solution at time t of (2.1) for the given u. The vector field X(x,u) is addressed as the generating field of g^t_u.

Example 2.2 Consider $dx_1/dt = x_2$, $dx_2/dt = -x_1 + u$. With u = 0, the flow g^t_0, generated by the uncontrolled vector field $x_2\partial/\partial x_1 - x_1\partial/\partial x_2$, is constituted by the group SO(2) of rigid clockwise rotations in the plane R^2.

Definition 2.3 Let D be a compact subset of R^n . Then, for a given u, the image at time t of D under the flow of X, $g^t_u(D)$ is defined as :

$$g^t_u (D) = \{ x \in R^n : x = g^t_u x_0 \text{ for some } x_0 \in D \} \qquad (2.2)$$

Definition 2.4 Let u be a fixed control function, the divergence of the vector field X(x,u) is defined as :

$$\text{div } X(x,u) = \text{Trace } [\partial X/\partial x] = \Sigma_{i=1}^{n} \partial X_i/\partial x_i \qquad (2.3)$$

Example 2.5 In example 2.2 above, the divergence of the uncontrolled vector field is zero (this means area invariance, under the flow map, of the evolution of compact regions in the plane. See [5, pp. 69, Theorem 2, Also 4, pp. 198, Corollary 1]).

<u>Theorem 2.6</u> (Stonger version of Liuoville's Theorem, [4, pp. 198]) Let D be a compact region in R^n with volume $V(0)$. If $V(t)$ denotes the volume of $g^t{}_u$ (D) (:= D(t)), for some given u, then at any time τ :

$$(dV/dt)|_\tau = \int_{D(\tau)} \text{div } X(x,u) \tag{2.4}$$

<u>Proof</u>. The proof of this theorem can be found in [5, pp. 69]. □

Let D be a compact region of R^n whose boundary, denoted by ∂D, is a smooth n-1 dimensional submanifold of R^n characterized by :

$$\partial D = \{ x \in R^n : s(x) = 0 \} \tag{2.5}$$

where $s : R^n \to R$ is a smooth function with nonzero gradient vector, $\partial s/\partial x$, almost everywhere on ∂D. We assume that ∂D is oriented in such a way that $s(x) < 0$ describes the bounded interior of D while $s(x) > 0$ is the open unbounded complement of D. The vector field ds will denote the unit outer normal vector field of ∂D, i.e., $\partial s/\partial x = || \partial s/\partial x ||\, ds$.

<u>Theorem 2.7</u> (The Divergence Theorem, [6, pp. 151])

$$\int_D \text{div } X(x,u) = \int_{\partial D} \langle ds, X(x,u) \rangle \tag{2.6}$$

where $\langle \cdot , \cdot \rangle$ denotes inner product.

<u>Definition 2.8</u> The flow map $g^t{}_u$ is locally a <u>contraction</u> on a given compact region D if there exists a small positive scalar ε such that $D \supset g^t{}_u(D)$ for all $0 < t < \varepsilon$. Conversely, $g^t{}_u$ is locally an <u>expansion</u> on D if $g^{-t}{}_u$, the inverse map of $g^t{}_u$, is a contraction on D (i.e., $D \supset g^{-t}{}_u(D)$, or, equivalently, $g^t{}_u(D) \supset D$).

<u>Example 2.9</u> Consider a disk D of radius r in R^2 . The flow $g^t{}_u$ generated by the vector field : $[x_2 - x_1(x_1^2+x_2^2 - u)]\partial/\partial x_1 + [-x_1 - x_2(x_1^2 + x_2^2 - u)]\partial/\partial x_2$ with $u = a^2 = constant < r^2$, is locally a contraction on D. On the other hand, $g^t{}_u$ is locally an expansion on D for $u = b^2 = constant > r^2$. (See Figures 1a and 1b)

<u>Theorem 2.10</u> Let X(x,u) be the generating field of $g^t{}_u$. Then, $g^t{}_u$ is locally a contraction (expansion) on D if and only if for all $x \in \partial D$, $\langle ds, X(x,u) \rangle < 0$ (> 0) .

<u>Proof</u> We shall only prove the contraction part in the theorem. The expansion part follows by similar arguments.

Let $g^t{}_u$ be locally a contraction on D, then for each $x \in \partial D$, the inner product : $\langle ds, g^\varepsilon{}_u(x) - x \rangle < 0$, for any arbitrarily small ε. Substituting $g^\varepsilon{}_u(x)$ by its series expansion about x, $g^\varepsilon{}_u(x) = x + \varepsilon X(x,u) +$ h.o.t , one finds: $\varepsilon \langle ds, X(x,u) \rangle + o(\varepsilon^2) < 0$, which holds true for an arbitrarily small ε if and only if $\langle ds, X(x,u) \rangle < 0$. To prove sufficiency, let $\langle ds, X(x,u) \rangle < 0$ for all $x \in \partial D$ but suppose that $g^t{}_u(D)$ is not entirely contained in D, i.e., $g^t{}_u$ is not a contraction. Then there exists at least one open region of ∂D which has a nonempty intersection with $g^t{}_u(D)$. Take any x in $g^t{}_u(D) \cap \partial D$. For a sufficiently small $\varepsilon > 0$, $\langle ds, g^\varepsilon{}_u(x) - x \rangle > 0$ (See Figure 2). Using again, $g^\varepsilon{}_u(x) = x + \varepsilon X(x,u) +$ h.o.t, in the inner product one concludes that $\varepsilon \langle ds, X(x,u) \rangle + O(\varepsilon^2) > 0$, i.e., $\langle ds, X(x,u) \rangle > 0$ on an open region of ∂D. This is a contradiction. ◻

The proof of the following corolary is an immediate consequence of Theorem 2.10 and the Divergence Theorem 2.7 above.

Corollary 2.11 Let g^t_u be locally a contraction (expansion) on D. Then, $dV/dt\,|_{t=0} < 0$ (> 0) . i.e., $\int_{\partial D} \langle ds, X(x,u)\rangle = \int_D \operatorname{div} X(x,u) < 0$ (> 0). □

2.2 Conditions for existence of Sliding Modes on Compact Manifolds

Definition 2.12 A Variable Structure Control law, with discontinuity surface ∂D, is a specification of a feedback control policy u(x), on (2.1), according to :

$$u(x) = \begin{cases} u^+(x) & \text{for } s(x) > 0 \\ u^-(x) & \text{for } s(x) < 0 \end{cases} \qquad u^+ \neq u^- \tag{2.7}$$

where one may assume, without loss of generality, that pointwise in x, $u^+(x) < u^-(x)$.

Definition 2.13 A global sliding regime (i.e., one existing everywhere except, possibly, on a set of measure zero) is said to exist on ∂D if and only if at every point $x \in \partial D$, the variable structure control law (2.7), acting on (2.1), is such that :

$$\lim_{s \to +0} L_{X(x,u^+)}\, s < 0 \iff \lim_{s \to +0} \langle ds, X(x,u^+)\rangle < 0$$
$$\lim_{s \to -0} L_{X(x,u^-)}\, s > 0 \iff \lim_{s \to -0} \langle ds, X(x,u^-)\rangle > 0 \tag{2.8}$$

where L_X denotes the Lie derivative (directional derivative) of the scalar function s with respect to the controlled vector field X.

Theorem 2.14 A sliding motion globally exists on ∂D if and only if, $g^t_{u^+}$ is a local contraction on D and $g^t_{u^-}$ is a local expansion on D. i.e, given a sufficiently small positive ε, for all $0 < t < \varepsilon$

$$D \supset g^t_{u^+}(D) \qquad \text{and} \qquad g^t_{u^-}(D) \supset D \tag{2.9}$$

Proof: Suppose a sliding regime globally exists on ∂D, then conditions (2.8) hold true. From Theorem 2.10 the set-inclusions (2.9) are also true. Suppose now (2.9) holds true. Then, using the results of theorem 2.10, one obtains on ∂D, $\langle ds, X(x,u^+(x))\rangle|_{x\in\partial D} = \lim_{s\to+0} \langle ds, X(x,u^+(x))\rangle < 0$. On the other hand, $\langle ds, X(x,u^-(x))\rangle|_{x\in\partial D} = \lim_{s\to-0}\langle ds, X(x,u^-(x))\rangle > 0$. Hence conditions (2.8) hold true and a sliding motion globally exists on ∂D. □

Example 2.15 Consider the disk D and the dynamical system of example 2.9. A global sliding motion exists on the circumference ∂D when the switching logic : $u = u^+(x) = a^2 < r^2$ for $x_1^2 + x_2^2 - r^2 > 0$ and $u = u^-(x) = b^2 > r^2$ for $x_1^2 + x_2^2 - r^2 < 0$, is used. (See Figure 3).

Corollary 2.16 If a sliding regime globally exists on ∂D, then

$$\int_D \operatorname{div} X(x,u^+(x)) < 0 \qquad \text{and} \qquad \int_D \operatorname{div} X(x,u^-(x)) > 0 \qquad (2.10)$$

Proof: Suppose a sliding regime globally exists on ∂D, then from (2.8), for all $x \in \partial D$, $\langle ds, X(x,u^+(x))\rangle < 0$ and $\langle ds, X(x,u^-(x))\rangle > 0$ hold valid. Taking the surface integral, over ∂D, of the inner products and using the Divergence Theorem 2.7 on each case, conditions (2.10) follow. □

Example 2.17 Consider a DC to DC power converter of the Boost type, shown in Figure 4. ([7]) .

$$dx_1/dt = b - w_0x_2 + u\, w_0x_2 = X_1(x,u)$$
$$dx_2/dt = w_0x_1 - w_1x_2 - u\, w_0x_1 = X_2(x,u) \qquad (2.11)$$

where $x_1 = \sqrt{L}\, I$, $y_2 = \sqrt{C}\, V$, $b = E/\sqrt{L}$, $w_0 = 1/\sqrt{LC}$, $w_1 = 1/RC$ and u denotes the switch position function, acting as a control input, which takes values in the discrete set U = { 0,1 }. We wish to know wheather or not, using a suitable switching policy, harmonic motions are possible for the Boost converter responses (DC to AC conversion) i.e., D is to be taken as a disk of radius r, centered at the origin, ∂D is the bounding circumference. An evaluation of the necessary conditions (2.10) leads to:

$$\int_D \text{div } X(x,1) = -\pi r^2 w_1 < 0 \quad , \quad \text{and} \quad \int_D \text{div } X(x,0) = -\pi r^2 w_1 < 0 \qquad (2.12)$$

which readily reveals that a global sliding motion does not exist on ∂D for the available control inputs in the discrete set U. As a matter of fact, a sliding motion does not exist on any nontrivial circumference in R^2. □

2.3 Characterization of the Ideal Sliding Dynamics and the Equivalent Control

Definition 2.18 (Hale, [8, pp. 266]). Let $s(x) = 0$ be a smooth manifold in R^n. We say that $s(x) = 0$ is a global integral manifold for the controlled system (2.1) if for certain smooth control function, $u(x)$, the state trajectories that start anywhere on $s(x) = 0$ remain on $s(x) = 0$ for all time. i.e., for each $x \in \partial D$, $g^t_u(x) \in \partial D$ for all $t > 0$.

Theorem 2.19 The compact manifold ∂D is an integral manifold of (2.1), for a given smooth u, if and only if $g^t_u(\partial D) = \partial D$ for all $t > 0$.

Proof If ∂D is an integral manifold of (2.1) for some smooth u then, by definition of integral manifold, for each $x \in \partial D$, and all t, $g^t_u(x) \in \partial D$ i.e., $\partial D \supset g^t_u(\partial D)$ for all t. Suppose now that $g^t_u(\partial D)$ does not contain ∂D for some t, then there exist open sets in ∂D which have an empty intersection with $g^t_u(\partial D)$. Taking any x on such an open set, one concludes that $g^t_u(x) \notin \partial D$ i.e., ∂D is not a global integral manifold for (2.1). This is a contradiction. Hence, $g^t_u(D) \supset \partial D$ for all t. From the double inclusion just shown, it follows that $g^t_u(\partial D) = \partial D$. Sufficiency is obvious. □

If a global sliding motion exists on ∂D then the average trajectories of (2.1) can be defined as ideally constrained to ∂D under the action of certain smooth control function known as the equivalent control, [1] and denoted by $u^{EQ}(x)$ with $x \in \partial D$. The equivalent control associated to a sliding regime is thus defined as a smooth state feedback control function, $u^{EQ}(x)$, for which the global sliding manifold ∂D becomes an integral manifold of (2.1). The tangency of the average trajectories to ∂D is characterized by the following manifold invariance condition, satisfied by the ideally smoothly controlled vector field $X(x,u^{EQ}(x))$:

$$L_{X(x,u^{EQ}(x))}\, s = 0 \quad \text{on } s = 0 \quad \text{i.e.,} \quad \langle\, ds,\, X(x,u^{EQ}(x))\,\rangle|_{s=0} = 0 \qquad (2.13)$$

If an equivalent control satisfying (2.13) is known, <u>the ideal sliding dynamics</u> is obtained by formally substituting u by $u^{EQ}(x)$ in (2.1). One obtains,

$$dx/dt = X(x,u^{EQ}(x)) \ , \quad x \in \partial D \qquad (2.14)$$

as the idealized description of the average trajectories, of the variable structure controlled system, on ∂D. This is the basis of the <u>method of the equivalent control</u> ([1, Ch. 2]).

By definition of the equivalent control and Theorem 2.19 above, it follows that for all t :

$$g^t{}_{u^{EQ}}\,(\partial D) = \partial D \qquad (2.15)$$

In general, for controlled vector fields of the form $X(x,u)$, equation (2.13), or (2.15), do not uniquely define the equivalent control (this topic is considered at length in [1, pp. 64-66]) except in some special cases (typically, when the controlled vector field is of the linear-in-the-control form: $X(x,u) = f(x)+ u\, g(x)$, provided the transversality condition $\langle ds,g \rangle \neq 0$ is satisfied [7]).

It has been shown for the linear in the control case that a necessary and sufficient condition for the existence of a sliding regime on $s(x) = 0$ (See [1, pp. 119] and [9]) is the existence and uniqueness of the equivalent control, pointwise bounded within the extreme feedback laws i.e., $u^+(x) < u^{EQ}(x) < u^-(x)$.

The invariance conditions (2.13),(2.15) are easily seen to be equivalent to an invariance condition on the evolution of the volume of $g^t{}_{u^{EQ}(x)}(D)$ for all t :

$$dV/dt|_{t=\tau} = \int_{D(\tau)} \text{div } X(x, u^{EQ}(x)) = 0 \qquad (2.16)$$

Existence of a smooth feedback control, $u^{EQ}(x)$, such that the volume invariance condition (2.16) is satisifed constitutes only a necessary, but not sufficient, condition for the existence of an equivalent control associated to a sliding regime on ∂D. This is established in the next corollary.

Corollary 2.20 If an equivalent control exists globally on ∂D, then (2.16) holds true, i.e., the volume of D remains constant under $g^{t}{}_{u}EQ$.

Proof: By definition of equivalent control, $\langle ds, X(x, u^{EQ}(x)) \rangle = 0$ at all times. From the Divergence Theorem 2.7, Theorem 2.6, Theorem 2.19, and the fact that the boundary of $g^t{}_u(D)$ equals the image of the boundary of D, under $g^t{}_u$, i.e., $\partial[g^t{}_u(D)] = g^t{}_u(\partial D)$, it follows that for any $\tau \geq 0$:

$$dV/dt|_{t=\tau} = = \int_{g^{\tau}_{u}EQ(D)} \text{div } X(x, u^{EQ}(x)) = \int_{\partial[g^{\tau}_{u}EQ(D)]} \langle ds, X(x, u^{EQ}(x)) \rangle$$

$$= \int_{g^{\tau}_{u}EQ(\partial D)} \langle ds, X(x, u^{EQ}(x)) \rangle = \int_{\partial D} \langle ds, X(x, u^{EQ}(x)) \rangle = 0. \quad \square \qquad (2.17)$$

The equivalent control forces the flow map $g^t{}_uEQ_{(x)}$ to preserve the volume of the region D. A sufficient condition to have (2.17) valid is that the subintegral quantity becomes zero, i.e., $\text{div } X(x, u^{EQ}(x)) = 0$. This leads to a first order quasilinear partial differential equation of the form:

$$\Sigma^n{}_{i=1} \{ \partial X_i(x, u^{EQ})/\partial x_i + [\partial X_i(x, u^{EQ})/\partial u^{EQ}](\partial u^{EQ}/\partial x_i) \} = 0 \qquad (2.18)$$

from where an $u^{EQ}(x)$ may be found.

Example 2.21 In example 2.15, an equivalent control may be obtained, using (2.18), as a solution of : $\text{div } X(x, u^{EQ}(x)) = -4(x_1{}^2 + x_2{}^2) + 2u^{EQ} + x_1 \partial u^{EQ}/\partial x_1 + x_2 \partial u^{EQ}/\partial x_2 = 0$. It is easily verified that $u^{EQ}(x) = x_1{}^2 + x_2{}^2$ is such a solution. Hence $u^{EQ}(x)|_{x \in \partial D} = x_1^2 + x_2^2 = r^2$. □

A corollary to Liouville's theorem ([5, pp. 69, also 4, pp. 198]) explicitly states that flow maps generated by Hamiltonian vector fields preserve the volume of compact regions. The ideal sliding dynamics $dx/dt = X(x,u^{EQ}(x))$, enjoys the same property, as established by Corollary 2.20, when applied to the particular region D bounded by the compact sliding manifold ∂D. Notice, however, that this <u>does not</u> mean that the ideal sliding dynamics of VSS undergoing sliding motions on compact manifolds are necessarily represented by Hamiltonian systems. The corollary to Liouville's theorem is a necessary, but not sufficient, condition for a system to be Hamiltonian.

<u>Example 2.22</u> To illustrate that the invariance condition (2.16) is merely a necessary condition for the existence of an equivalent control associated to a sliding motion, consider the problem of finding $u^{EQ}(x)$ for the case of example 2.17. This control must render smooth oscillatory responses of harmonic nature on (2.11). For this, consider the sliding surface candidate $\partial D = \{ x \in R^2 : s(x) = x_1^2 + x_2^2 - r^2 = 0 \}$. Then :

$$\int_D \mathrm{div}\, X(x,u^{EQ}(x)) = \int_D \{x_2[\partial u^{EQ}/\partial x_1] - x_1[\partial u^{EQ}/\partial x_2] - w_1/w_0\} dx_1 dx_2 = 0 \quad (2.19)$$

A sufficient condition for (2.19) to be valid is that the subintegral quantity becomes zero. Hence, the following partial differential equation is to be satisfied by $u^{EQ}(x)$:

$$x_2 [\partial u^{EQ}/\partial x_1] - x_1[\partial u^{EQ}/\partial x_2] = w_1/w_0 \quad (2.20)$$

Equation (2.20) has as a solution, $u^{EQ}(x_1,x_2) = [w_1/w_0] \tan^{-1}[x_1 x_2^{-1}]$. A smooth feedback control thus exists which satisfies the invariance condition (2.16). However, it was shown that, in this case, a sliding motion does not exist globally on ∂D, hence the found $u^{EQ}(x)$ is not associated to a sliding regime.

4. CONCLUSIONS

A general geometric characterization for the existence of sliding regimes on compact manifolds for nonlinear variable structure feedback systems has been given. The characterization involves a set-theoretic inclusion condition generated by the control dependent flow map when applied on the compact region contained by the sliding manifold. A sign condition on the volume integral of the divergence of the generating controlled vector field is derived as a necessary, but not sufficient, condition for the existence of a sliding motion. The invariance conditions, or ideal sliding conditions, are characterized in terms of volume-preserving evolution of the flow map associated with the ideal sliding dynamics defined on the proposed sliding manifold. An application of the general results to periodic sliding motions in R^2 was illustrated via several simple examples. A generalization of the obtained results to the case of noncompact manifolds is by no means trivial and constitutes an area for further research.

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FIGURE CAPTIONS

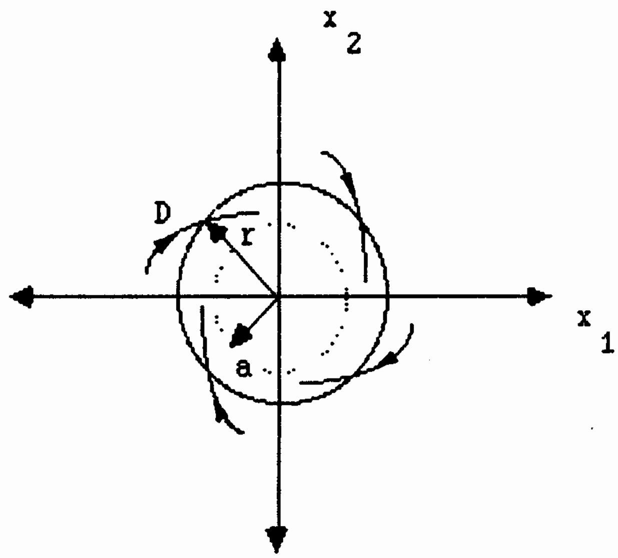

Figure 1a. A local contraction on D

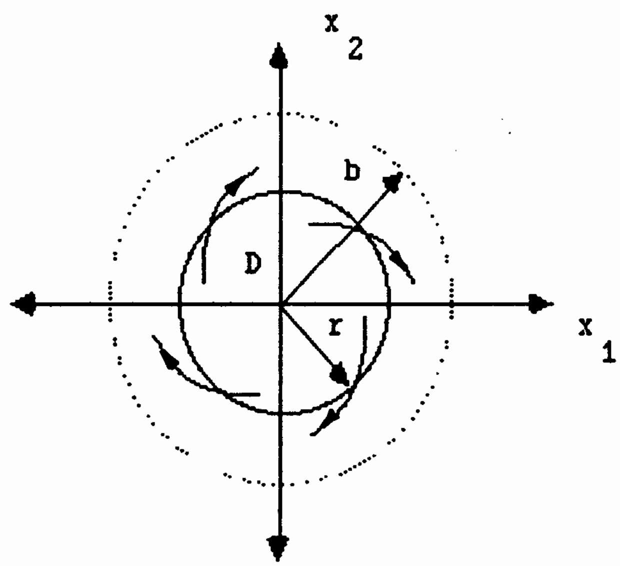

Figure 1b. A local expansion on D.

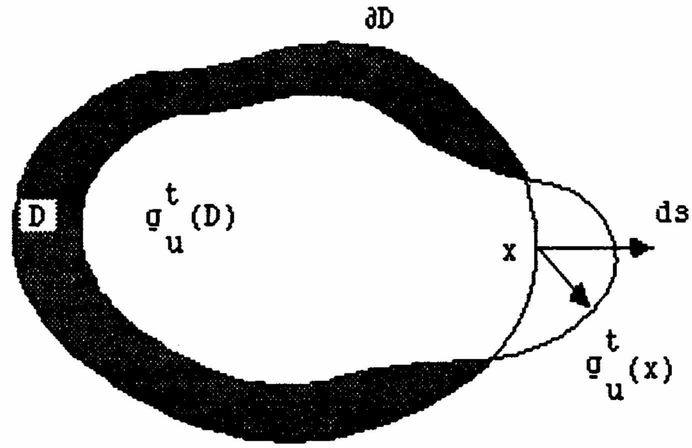

Figure 2. Proof by contradiction of Theorem 2.10

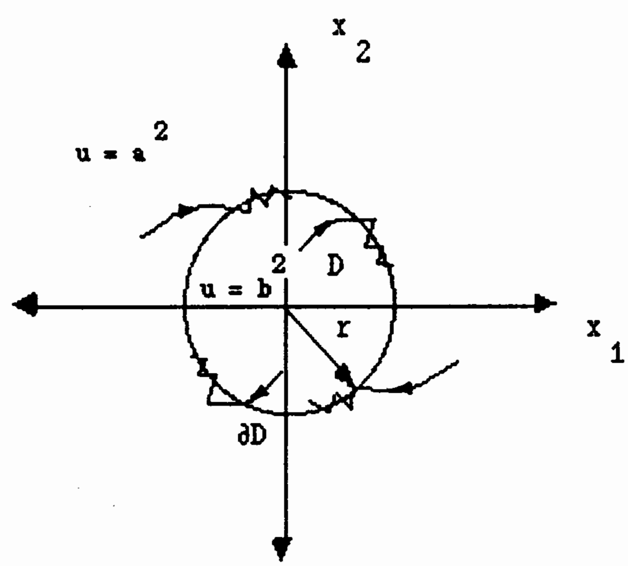

Figure 3. Periodic Sliding Motion in R^2

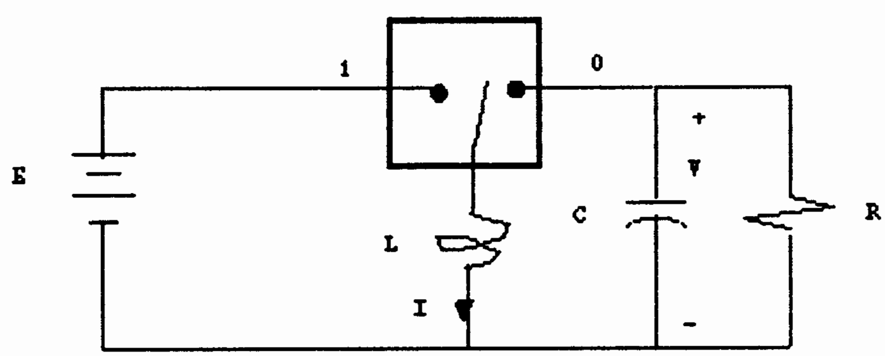

Figure 4 : Boost Converter