

See-saw and Supersymmetry or Exact R-parity

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We show how the renormalizable see-saw mechanism in the context of supersymmetry and spontaneously broken B–L symmetry implies exact R-parity at all energies. We argue that supersymmetry plays an important role in providing a “canonical” form for the see-saw, in particular in grand unified theories that solve the doublet-triplet splitting problem via the Dimopoulos-Wilczek mechanism.

A. The see-saw mechanism The see-saw mechanism [1] is a natural and simple way of understanding the smallness of neutrino mass. If one adds a right-handed neutrino ν_R to the Standard Model (SM), the renormalizable interactions generate a small neutrino mass

$$m_\nu = \frac{m_D^2}{m_{\nu_R}} \quad (1)$$

where m_D is the neutrino Dirac mass term and m_{ν_R} is the Majorana mass of ν_R . Since we expect the gauge singlet mass m_{ν_R} to be large, $m_{\nu_R} \gg M_W$, (1) tells us that $m_\nu \ll m_D$. The solar and atmospheric neutrino data strongly suggest small neutrino masses below 1 eV [2]. For generic m_D of the order of charged lepton or quark masses this points to m_{ν_R} bigger than 10^{11} GeV or so (and possibly as large as the GUT scale). We stick to this in the rest of the paper.

Here we address the issue of the see-saw mechanism in the context of supersymmetry. This is an extremely important question for at least two reasons. First, a central issue of the supersymmetric standard model is the fate of baryon and lepton numbers. The conservation of B and L is normally connected to R-parity, and it would be of great use to have a more fundamental, underlying principle settling this question. Second, broken R-parity implies nonvanishing neutrino masses [3,4] and hence obscures the see-saw predictions. It is here that the left-right (LR) symmetry [5] (or B–L gauge invariance) plays an important role: it implies R-parity in the underlying theory [6]. Namely, R-parity can be written as

$$R = (-1)^{3(B-L)+2s} \quad (2)$$

This means that we must break B–L spontaneously in order to generate the right-handed neutrino mass. We assume that this happens through the renormalizable interactions of right-handed neutrinos with a B–L=2 field (singlet under the SM), in order to avoid the forbidden breaking of R-parity at high energies $\sim m_{\nu_R}$. We refer to this as the renormalizable see-saw.

Of course, it still remains to be proved that in the process of coming down to low energies R-parity is not

broken. We carefully address this issue below and prove the following theorem:

The renormalizable supersymmetric see-saw mechanism in theories with B–L symmetry, local or global, implies an exact R-parity even in the low energy effective theory.

This remarkable statement is a simple extension of the impossibility of breaking R-parity spontaneously in the MSSM, and it has important phenomenological and cosmological consequences. For example, exact R-parity ensures the stability of the lightest supersymmetric partner, a natural dark matter candidate.

Thus, the see-saw mechanism plays a useful role in the determination of the structure of the supersymmetric Standard Model (SSM). In turn, as we discuss below, supersymmetry also helps determine the precise form of the see-saw. Namely, in general in theories beyond the Standard Model the canonical form as defined in (1) is not complete. We show that supersymmetry may guarantee in some instances the canonical form, as for example in grand unified theories which solve the hierarchy problem by the missing VEV mechanism [7].

B. See-saw and exact R-parity The argument here is very simple and is a generalization of our recent work on supersymmetric LR theories [8].

Let us assume that the original theory possesses a $U(1)_{B-L}$ symmetry, and the see-saw is achieved by renormalizable terms only. Then m_{ν_R} must be induced through the VEV $\langle \sigma \rangle$, where σ is a B–L=2 superfield. Anomaly cancellation requires the existence of a B–L=-2 superfield $\bar{\sigma}$. The superpotential is then given by

$$W_R = \frac{1}{2} f N^2 \sigma + g(\sigma \bar{\sigma}) \quad (3)$$

where N is the B–L=-1 singlet superfield which contains a right-handed neutrino $\nu^c \equiv C \bar{\nu}_R^T$, and $g(\sigma \bar{\sigma})$ is some function of the B–L invariant combination $\sigma \bar{\sigma}$.

By properly choosing $g(\sigma \bar{\sigma})$, a non-vanishing $\langle \sigma \rangle \neq 0$ can be enforced [9].

From

$$F_N = f N \sigma = 0 \quad (4)$$

it is clear that $\langle N \rangle = \langle \nu^c \rangle = 0$ in our vacuum [10]. Notice that for this to be valid all that we need is that σ has a

non-vanishing VEV, and thus this important result does not depend on the details of the model. In particular, it holds true in any supersymmetric LR or SO(10) theory with a renormalizable see-saw. In short, at the scale M_R R-parity remains unbroken, as it should be.

Now what happens as one descends in energy all the way to M_W ? The see-saw and supersymmetry guarantee a large mass for $\tilde{\nu}^c$: $m_{\tilde{\nu}^c} = m_{\nu^c} = fM_R$ and thus the vacuum $\langle \tilde{\nu}^c \rangle = 0$ is stable against any perturbation due to soft supersymmetry breaking terms. However, the same is not true of $\tilde{\nu}$, since it is massless at the scale M_R . The question of the fate of R-parity is simply a question as to whether $\langle \tilde{\nu} \rangle$ vanishes or not (of course, $\langle \tilde{\nu} \rangle \neq 0$ would trigger a VEV for $\tilde{\nu}_c$ through linear terms, but this effect is negligible). To see what happens, let us recall first the situation with the minimal supersymmetric Standard Model (MSSM).

We define the MSSM as the SSM without R-parity breaking terms. Then, a nonvanishing VEV for $\langle \tilde{\nu} \rangle$ implies the existence of a “doublet” [3] Majoron [11], the Goldstone boson associated with the spontaneous breaking of the continuous lepton number. Such a Majoron is ruled out experimentally [12]. The point is that $\langle \tilde{\nu} \rangle$ must be small. Consider the most conservative case when $\tilde{\nu}$ is $\tilde{\nu}_\tau$. Since ν_τ mixes with gauginos through $\langle \tilde{\nu}_\tau \rangle$, one obtains an effective mass

$$m_{\nu_\tau} \simeq \langle \tilde{\nu}_\tau \rangle^2 / m_\lambda \quad (5)$$

where m_λ is a gaugino mass which should lie below 1 TeV. From the experimental limit on τ neutrino mass one gets an upper limit [13] $\langle \tilde{\nu}_\tau \rangle \leq 10$ GeV. Furthermore, if one believes that the solar and atmospheric neutrino puzzles are explained by the usual three neutrinos, one gets a much better limit $m_{\nu_\tau} \leq 5$ eV [14] implying $\langle \tilde{\nu}_\tau \rangle \leq 10$ MeV. Now, the scalar partner R of the (pseudoscalar) Majoron, J , has a mass of the order of $\langle \tilde{\nu}_\tau \rangle$ and thus one would have the forbidden decay $Z \rightarrow J + R$. In the MSSM, it is simply impossible to break R-parity spontaneously. It is broken explicitly or not at all.

Strictly speaking, in the MSSM there is a much more stringent limit on $\langle \tilde{\nu}_\tau \rangle$ from astrophysical considerations. Unless $\langle \tilde{\nu}_\tau \rangle \leq 100$ keV, the Majoron would be produced in stars too copiously and radiate their energy away [15,3].

Let us now see what happens in the see-saw case. We have shown that $\langle \tilde{\nu}^c \rangle = 0$, and unless $\langle \tilde{\nu} \rangle \neq 0$ there will be no breaking of R-parity whatsoever. Now, once all the fields with mass $\sim M_R$ are integrated out the effective theory is the MSSM with all the effects of the large scale suppressed, i.e. MSSM + $O(1/M_R)$ effects, as dictated by the decoupling theorem. One obtains an effective operator in the superpotential

$$W_{\text{eff}} = \frac{(LH)^2}{M_R} \quad (6)$$

where L and H are the lepton doublet and one of the Higgs superfields, respectively. The soft term $m_s W_{\text{eff}}$ generates a tiny mass for the Majoron

$$m_J^2 \simeq \frac{m_s M_W^2}{M_R} \quad (7)$$

(recall that the Majoron is predominantly the imaginary component of the $\tilde{\nu}$ field). Since $m_J \ll M_Z$, we end up with the same prediction of the ruled-out Z -decay into $J + R$. Surprisingly enough, much as in the MSSM, R-parity remains an exact symmetry at all energies. As such, the argument is independent as to whether B–L is a local or a global symmetry.

All of the discussion above applies to realistic theories. As we have seen, in order to have a theory of R-parity, one needs to assume B–L symmetry, and this happens automatically in any theory based on LR symmetry. Symmetry breaking leading to renormalizable see-saw has been studied extensively in [8].

C. Supersymmetry and the canonical see-saw form

In non-supersymmetric theories with LR symmetry one cannot ascribe the “canonical” form given in (1) to the see-saw. In these theories σ must be a triplet Δ_R of the $SU(2)_R$ gauge group, and the LR symmetry implies the existence of the $SU(2)_L$ triplet Δ_L . A simple analysis shows that it must have a VEV too [16]

$$\langle \Delta_L \rangle = \alpha \frac{M_W^2}{M_R} \quad (8)$$

where α is an unknown ratio of the quartic couplings in the potential. This stems from couplings

$$\lambda \Delta_L \Phi^2 \Delta_R \quad (9)$$

where Φ is the Higgs multiplet responsible for the fermionic Dirac mass terms. One cannot forbid such a term while preserving the see-saw, since it is logarithmically divergent at one loop [16], as shown in diagram (a) of Fig. 1 below.

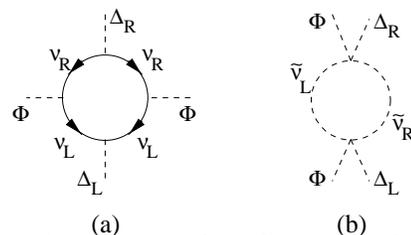


FIG. 1. (a) An infinite one-loop diagram for the interaction $\Delta_L \Phi^2 \Delta_R$, and (b) its supersymmetric counterpart.

Since Δ_L and Δ_R are coupled to neutrinos in a LR symmetric manner

$$\mathcal{L}_Y = f(\ell_L^T C \Delta_L \ell_L + \ell_R^T C \Delta_R \ell_R) \quad (10)$$

where ℓ_L is the leptonic doublet, the nonvanishing $\langle \Delta_L \rangle$ provides a direct mass term to left-handed neutrinos

$$m_\nu = f \langle \Delta_L \rangle - \frac{m_D^2}{m_{\nu_R}} \quad (11)$$

It is still of the see-saw form, *i.e.* proportional to $1/M_R$, but it obscures the popular canonical form (1) (sometimes referred to in the literature [17,18] as type I see-saw, while the see-saw formula with the additional piece generated by the VEV of Δ_L in (11) is called type II).

A possible way out of this problem is to break LR symmetry at a much larger scale than M_R through parity-odd singlets [19] and suppress $\langle \Delta_L \rangle$.

Another possibility is simply to supersymmetrize the theory. In supersymmetry no such contribution exists; diagram (a) is canceled out by its supersymmetric counterpart, diagram (b) in Fig. 1. In order to generate the term in (9) supersymmetry must be broken. It is easy to estimate then this effective term which arises from the difference in right-handed neutrino and sneutrino masses. For the low-energy supersymmetric theory $m_{\nu_R}^2 \simeq m_{\nu_R}^2 + M_W^2 = f^2 M_R^2 + M_W^2$ and for $M_R \gg M_W$, we obtain

$$\lambda \simeq f^2 \sum_g \left(\frac{m_D}{M_W} \right)^2 \ln \frac{m_{\nu_R}^2}{m_{\nu_R}^2} \simeq \sum_g \left(\frac{m_D}{M_R} \right)^2 \quad (12)$$

where \sum_g is a sum over generations, and for simplicity f is taken to be generation-independent. By adding an effective mass term $m^2 \Delta_L^2$ to the term (9), one gets upon minimization an order of magnitude estimate

$$\langle \Delta_L \rangle \simeq \left(\frac{M_W}{m} \right)^2 \sum_g \frac{m_D^2}{M_R} \quad (13)$$

Notice the factor $(M_W/m)^2$ compared to the canonical see-saw formula (1). The size of m is model dependent. In supersymmetric models based on renormalizable interactions [8] m is of order M_R in which case the suppression factor is enormous so that the canonical form (1) is recovered. In models based on non-renormalizable interactions, though, one has instead [8] $M_R^2 = mM$, where M is the cut-off of the theory. For $M \simeq M_{Pl}$ one has

$$\langle \Delta_L \rangle = \sum_g \frac{m_D^2}{M_R} \left(\frac{M_W^2 M_{Pl}^2}{M_R^4} \right) \quad (14)$$

Only for M_R at its minimum expected value $\simeq 10^{11}$ GeV this term can compete with the canonical one, but it becomes rapidly negligible with growing M_R .

However, this is not the whole story. One can have corrections to the canonical see-saw even without supersymmetry breaking from possible non-renormalizable term in the *superpotential* of the form

$$\frac{1}{M} \Delta_L \Phi^2 \Delta_R^* \quad (15)$$

Such terms could naturally arise from Planck scale effects with $M = M_{Pl}$ or, more interestingly, from the GUT scale physics.

For example, in the minimal renormalizable supersymmetric SO(10) theory M is simply M_X , the scale of SO(10) breaking. In this model $M_X \simeq \langle S \rangle$, where S is the symmetric 54-dimensional representation, and Δ_L and Δ_R belong to a 126-dimensional representation Σ . From the superpotential interactions (Φ is the light Higgs, usually in the 10-dimensional representation)

$$W = \Phi^2 S + \Sigma^2 S \quad (16)$$

after integrating out the heavy field S , one gets the effective non-renormalizable interaction (15), with $M \simeq M_X$. $\langle S \rangle$ breaks SO(10) down to the Pati-Salam group $SU(2)_L \times SU(2)_R \times SU(4)_c$, which is next broken by a 45-dimensional representation A , with $\langle A \rangle = M_{PS}$. It can also be easily shown that $m \simeq M_{PS}$.

To find the VEV of Δ_L we set the F-terms (see [8] for details) to zero to get

$$\langle \Delta_L \rangle \approx \frac{\langle \Phi \rangle^2 \langle \Delta_R \rangle}{mM} \equiv \epsilon \frac{\langle \Phi \rangle^2}{\langle \Delta_R \rangle} \quad (17)$$

where $\epsilon = M_R^2 / M_{PS} M_X$. We have found [20] that although a number of new light states appear, successful unification tends to push the intermediate scales towards the same (GUT) scale, giving ϵ anywhere between 1 and 10^{-4} .

The light neutrino mass thus gets the following form:

$$m_\nu \approx (f^2 \epsilon - h_D^2) \frac{\langle \Phi \rangle^2}{m_{\nu_R}} \quad (18)$$

where f is the coupling of Δ_L to left neutrinos and h_D is the neutrino Yukawa coupling, and ϵ is in general model-dependent. Even for $\epsilon \sim 10^{-4}$ the non-canonical part cannot be considered small, since for the first two generations the canonical see-saw has a strong suppression due to the smallness of m_D .

Notice that the undesired operator (15) originates from the exchange of a heavy (3,3,1) field (in the $SU(2)_L \times SU(2)_R \times SU(4)_c$ notation), the only field that can couple to both Φ and Δ_L, Δ_R . We recover the canonical see-saw by choosing a GUT scale Higgs that does not contain this field, for example the 210 representation of SO(10). It is noteworthy that 210 contains a parity-odd singlet, and thus can give a canonical see-saw even in the non-supersymmetric case (if $M_R \ll M_X$) [19].

Another possibility is that even if the GUT scale Higgs contains a (3,3,1) field, one forbids its coupling to Δ_R and Δ_L by a discrete symmetry. This is exactly what happens in the Dimopoulos-Wilczek missing VEV mechanism [7] that solves the doublet-triplet splitting problem. In this case, there is no effective non-renormalizable interaction (15) (modulo $1/M_{Pl}$ terms expected to be small

for $M_R \leq M_X$). An example of an SO(10) model that utilizes the Dimopoulos-Wilczek mechanism is given in [21] (see also [22]). In this theory the Higgs fields are in a pair of ten-dimensional representations Φ_1 and Φ_2 . The splitting is achieved with a 45-dimensional representation A and the superpotential

$$W = A\Phi_1\Phi_2 + S\Phi_2^2 \quad (19)$$

When S gets a VEV $\sim M_X$ and A a VEV $\text{diag}(a, a, a, 0, 0) \times \tau_2$, it is obvious that both Higgs triplets get heavy, while a doublet Higgs remains massless. The absence of the $S\Phi_1^2$ term precisely forbids the generation of the troubling non-renormalizable terms (15). Thus this solution to the doublet-triplet splitting problem leads to the canonical form for the see-saw (1).

Notice that the above argument could be invalidated if we were to use 16 and $\overline{16}$ instead of 126 in order to generate see-saw [18]. However, this choice would imply the unacceptable breaking of R-parity at high energies and the theory would require extra discrete symmetries, contrary to the spirit of this paper.

E. Summary and Outlook The fate of R-parity is probably the central issue of the MSSM. This paper connects it to the issue of neutrino mass. We show that *the renormalizable see-saw mechanism through the spontaneous breaking of B-L symmetry implies exact R-parity at all energies.*

On the other hand, in the SSM one could always attribute the small neutrino mass to the explicit, albeit small breaking of R-parity. What we have learned here is that this is completely orthogonal to the see-saw mechanism: if the see-saw is operative then simply R-parity never gets broken. This should be a welcome result to the practitioners of R-parity breaking, since this mechanism is then not obscured by the see-saw as the origin of neutrino mass.

The exact form of the see-saw is model dependent. The popular canonical form for see-saw in (1) can in general get additional see-saw terms as in (18) from higher-scales in the theory. However, in GUTs that solve the doublet-triplet splitting problem via Dimopoulos-Wilczek mechanism such extra terms are absent.

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