Gauge theories at finite temperature and the background field method

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The background field method has been employed to renormalize the real-time non-abelian field theory at finite temperature. The one-loop renormalization group equations for the gauge coupling are solved and its behaviour is obtained with respect to both temperature and momentum scale.

1. INTRODUCTION

Thermo field dynamics (TFD) [1][2][3] [4][5] is an elegant real-time formalism of quantum field theory at finite temperature in which the ground state is identified as the temperature-dependent vacuum and any statistical average is an expectation value on this vacuum. The background field method [6][7][8] [9][10][11], on the other hand, is a well-known technique for performing calculations in quantum field theories (at zero temperature), particularly in gauge theories [9][10][11] [12][13][14], which allows one to compute radiative corrections while maintaining gauge invariance. This technique also provides notorious simplifications, since no vertex corrections are to be performed. In this work, by the use of the background field method and TFD, a masless non-abelian gauge field theory at finite temperature is renormalized off-shell. As an application, the temperature and scale-dependent gauge coupling is obtained.

This work is organized as follows. In section 2 we present our results up to one loop and obtain the renormalization factors needed to derive the renormalization group equation (RGE) at finite temperature [4][5] for the effective coupling constant. In section 3 these RGEs are derived and explicitly integrated, obtaining the behaviour of the coupling constant with respect to both temperature and momentum scales. The conclusions and some comments on related works are given in section 4.

2. BACKGROUND FIELD CALCULATIONS OF THE RENORMALIZATION FACTORS

In the background field method we must renormalize only the background field B_μ, the coupling constant g and the gauge fixing parameter α. The quantum gauge field and the ghost field need not be renormalized, since they only appear inside loops. No vertex functions are to be considered [9][10][11]. In TFD all divergences are renormalized by the temperature-independent counterterms [4][5]. Since in the background field method these divergences are forced to take a gauge-invariant form, the background field renormalization factor Z_B and the coupling constant renormalization Z_g are related through

$$Z_g = Z_B^{-1/2}, \tag{1}$$

where

$$(B_\mu)_0 = Z_B^{1/2} B_\mu, \quad g_0 = Z_g \mu^\varepsilon g. \tag{2}$$

Therefore, we relate Z_g and Z_B through eq.(1), also at finite temperature.

In the one-loop approximation, the Feynman rules for the interaction vertices are the same as in the zero-temperature case, and therefore identical to those in ref. [9][10][11]. Our propagators are the same as in ref. [15][16].

The graphs required to evaluate up to one loop the renormalization factor Z_B, are shown in fig.1. To regularize and to renormalize the divergences, we employ dimensional regularization [17] and subtraction at space-like momenta p with $p^2 = -M^2$ and $p_0 = 0$.

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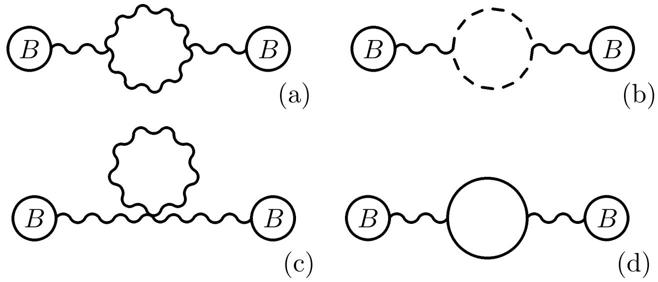

Fig. 1. Graphs for the one-loop calculation of the background field renormalization factor Z_B. Wavy lines are quantum gauge field propagators, wavy lines terminating in a "B" represent external gauge particles, solid lines are fermion propagators and dashed lines represent ghost propagators.

We carry out the renormalization in the Feynman-type gauge, $\alpha = 1$. Adding the contributions of graphs 1a-1d, we have

$$\begin{aligned}\Pi^{ab}_{\mu\nu}(p) = i\delta^{ab}\left(\frac{g}{4\pi}\right)^2 (g_{\mu\nu}p^2 - p_\mu p_\nu) \quad & \Bigg\{ C_A \left[+\frac{11}{3}\frac{1}{\hat{\varepsilon}} + \frac{67}{9} - \frac{1}{3}\left(\frac{2\pi}{\beta M}\right)^2 + 7F_0^-(\beta M) + F_2^-(\beta M) \right] \\ & + T_F \left[-\frac{4}{3}\frac{1}{\hat{\varepsilon}} - \frac{20}{9} - \frac{1}{3}\left(\frac{2\pi}{\beta M}\right)^2 + 2F_0^+(\beta M) + 2F_2^+(\beta M) \right] \Bigg\} \\ + i\delta^{ab}\left(\frac{g}{4\pi}\right)^2 g_{\mu 0}g_{\nu 0}p^2 \quad & \Bigg\{ C_A \left[-\left(\frac{2\pi}{\beta M}\right)^2 + 3F_0^-(\beta M) - 5F_2^-(\beta M) \right] \\ & + T_F \left[-\left(\frac{2\pi}{\beta M}\right)^2 - 10F_0^+(\beta M) + 6F_2^+(\beta M) \right] \Bigg\},\end{aligned} \tag{3}$$

where

$$\frac{1}{\hat{\varepsilon}} = \frac{1}{\varepsilon} - \gamma - \ln\frac{M^2}{4\pi\mu^2},$$

$$f^{abc}f^{dbc} = C_A\delta^{ab}, \qquad \mathrm{Tr}(T^aT^b) = T_F\delta^{ab}, \qquad (C_A = N,\ T_F = \frac{1}{2},\ \text{for SU(N)}),$$

$$F_n^{\pm}(\beta M) = \int_0^\infty dx \frac{x^n}{\exp(\frac{1}{2}\beta M x) \pm 1} \ln\left|\frac{1+x}{1-x}\right|, \tag{4}$$

$\beta = 1/T$ and T is the temperature.

From $\Pi^{ab}_{\mu\nu}$ we can read off the background field renormalization factorZ_B

$$\begin{aligned}Z_B - 1 = \ & C_A \left[+\frac{11}{3}\frac{1}{\hat{\varepsilon}} + \frac{67}{9} - \frac{1}{3}\left(\frac{2\pi}{\beta M}\right)^2 + 7F_0^-(\beta M) + F_2^-(\beta M) \right] \\ + \ & T_F \left[-\frac{4}{3}\frac{1}{\hat{\varepsilon}} - \frac{20}{9} - \frac{1}{3}\left(\frac{2\pi}{\beta M}\right)^2 + 2F_0^+(\beta M) + 2F_2^+(\beta M) \right]\end{aligned} \tag{5}$$

3. RGEs AT FINITE TEMPERATURE

The renormalized gauge coupling satisfies the following RGEs [4][5]

$$\begin{aligned}M\frac{\partial}{\partial M}g &= \frac{1}{2}gM\frac{\partial}{\partial M}\ln Z_B \\ &= -\frac{1}{(4\pi)^2}g^3C_A\left(\frac{11}{3} + \frac{1}{2}M\frac{\partial}{\partial M}\Theta(\beta M)\right) - \frac{1}{(4\pi)^2}g^3T_F\left(\frac{4}{3} + \frac{1}{2}M\frac{\partial}{\partial M}\zeta(\beta M)\right),\end{aligned} \tag{6}$$

$$\beta\frac{\partial}{\partial\beta}g = \frac{1}{2}g\beta\frac{\partial}{\partial\beta}\ln Z_B = -\frac{1}{(4\pi)^2}g^3C_A\left(\frac{1}{2}\beta\frac{\partial}{\partial\beta}\Theta(\beta M)\right) - \frac{1}{(4\pi)^2}g^3T_F\left(\frac{1}{2}\beta\frac{\partial}{\partial\beta}\zeta(\beta M)\right), \quad (7)$$

where

$$\Theta(\beta M) = \frac{1}{3}\left(\frac{2\pi}{\beta M}\right)^2 - 7F_0^-(\beta M) - F_2^-(\beta M) \quad (8)$$

and

$$\zeta(\beta M) = \frac{1}{3}\left(\frac{2\pi}{\beta M}\right)^2 - 2F_0^+(\beta M) - 2F_2^+(\beta M). \quad (9)$$

In the perturbative regime we can integrate (6) and (7) to obtain

$$g^2 = g_0^2\left[1 + 2C_A\left(\frac{g_0}{4\pi}\right)^2\left(\frac{11}{3}\ln\frac{M}{M_0} + \Theta(\beta M) - \Theta(\beta_0M_0)\right) + 2T_F\left(\frac{g_0}{4\pi}\right)^2\left(-\frac{4}{3}\ln\frac{M}{M_0} + \zeta(\beta M) - \zeta(\beta_0M_0)\right)\right]^{-1}. \quad (10)$$

The zero-temperature parts are in complete agreement with those encountered in the literature [9][10][11]. For fixed momenta, the gauge coupling shows a power-like dependence on the temperature. It also shows a similar behaviour to the gauge coupling renormalized at zero temperature only in the case when βM remains constant. Both results have been previously reported [15][16][18][19].

4. FINAL REMARKS

The background field method has been employed to renormalize the real-time finite temperature non-abelian gauge theory based on the TFD formalism, showing a high degree of simplification, as it does in gauge theories at zero temperature, in contrast to conventional methods [15][16][18][19]. However, our results in qualitative but not quantitative agreement with those encountered in previous works [15][16][18][19]. It is worth mentioning that this is also the case between ref. [15][16] and ref. [18][19]. For $\beta M \geq 4$, our results are close to those found in ref. [15][16] for the gauge coupling defined through the 3G vertex. This is not a trivial issue, because all the results obtained hitherto are dependent on the choice of vertex and of the subtraction point. Note also that we have not found the G_n terms which appear in ref. [18][19]. In these references, the covariant gauge is employed. Although the present calculation has been performed in the Feynman-type gauge, $\alpha = 1$, we have left open the gauge choice for the background field. We understand that all the obtained results (including the present one) rely heavily on the fact that the real-time formalisms of quantum field theory at finite temperature resemble those at zero temperature. The discordances found are to be revisited on somewhat more theoretical grounds. This subject is now under consideration. The extension of the present work up to the two-loop calculation is in progress.

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