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Two gravitationally Chern-Simons terms are too many

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Abstract

It is shown that topological massive gravity augmented by the triadic gravitational Chern-Simons first order term is a curved a pure spin-2 action. This model contains two massive spin-2 excitations. However, since its light-front energy is not semidefinite positive, this double CS-action does not have any physical relevance.In other words, topological massive gravity cannot be spontaneously broken down by the presence of the triadic CS term.

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Spin needs at least two space-like dimensions. Consequently, quantum dynamical effects for spinning particles requires space-times having dimensions not below three. If one belives that gravity is the self-interacting massless spin-two theory, then one has the uniform result that Einteins's action is the appropiate model in any dimension higher than three. For $d = 3$ one is faced with the fact that there are no massless excitations having (non zero) helicity [1]; if one believes that the essence of gravity stems in its spin (as we do), one has to consider massive spin-2 models, as it is the case either of topological massive gravity [2] or the recently proposed vector Chern-Simons gravity [3]. In $d = 3$ Einstein's action does not contain local physical excitations. It cannot be taken as the unique source of a pure spin-two theory. It is well known that however it leads to very interesting fully topological description [4] of gravity.

In spite of this seemingly profund difference between $3 - d$ Einstein's gravity and greater than three Einstein's gravities they share a uniform property: when a spin-two Fierz-Pauli mass term is added to them and goes to the associated rigid d-dimensional Minkowski space time, all of them provide a pure, (massive) spin-2 physical theory.

Since in $d = 3$ the physical spin-two theory, must be massive, it is quite straightforward to ask to all three dimensional models of gravity how they behave of they are broken down through a Fierz-Pauli mass term. Or more in general how they behave when some type of break-down mechanism is switched on the initial spin-2 system.

In this talk we shall concentrate on what happens to massive spin-two topological massive gravity (TMG) where instead of adding a Fierz-Pauli mass term a softer break-down mechanism is present: we shall analyse what happens to TMG wen a second Chern-Simons gravitational term breaks the initial local Lorentz invariance of TMG.

The interest of TMG goes beyond its academic possibility. It is one of the simplest non trivial models of a gravity theory having topological forms. It is well known that it is a distintive feature of effective string gravities that they contain terms having topological or quasi-topological structures like the ten-dimensional Lorentz-Chern-Simons [5] or the Gauss-Bonet [6] type of terms. So perhaps we might learn what are the qualitative changes arising from the presence of topological terms in effective realistic string gravities by analysing the novel properties (with respect to Einstein's) of $3 - d$ TMG.

The action of topological massive gravity S is the difference of the third order Lorentz-Chern-Simons term L and Einstein's action

$$S = L - Et = (2\mu)^{-1}\kappa^2 < \omega_p{}^a \epsilon^{prs}\partial_r \omega_{sa} - 2(3)^{-1}\epsilon^{prs}\epsilon_{abc}\omega_p{}^a\omega_r{}^b\omega_s{}^c >$$
$$-\kappa^{-2} < \omega_p{}^a \epsilon^{prs}\partial_r e_{sa} - 2^{-1} e_p{}^a \epsilon^{prs}\epsilon_{abc}\omega_r{}^b\omega_s{}^c > . \quad (1)$$

First latin letters denote Lorentz indices, middle of the alphabet world indices, $\eta^{00} = \eta^{11} = \eta^{22} = +1$. $\omega_p{}^a, e_{sa}$ are non independent quantities. It is understood that the affinities $\omega_p{}^a$ are determined in the standard way to be torsionless by their associated field equations:

$$\epsilon^{pqr}(\delta_q e_{ra} - \omega_q{}^c \epsilon_{cba} e_r{}^b) = 0. \quad (2)$$

The Lorentz CS-term is locally conformal, Lorentz and diffeomorphism invariant while Einstein's action is not locally conformal invariant.

S can be regarded as the result of having spontaneously broken the local conformal invariance of L by means of Einstein's action. S has a unique degree of freedom having a pure massive spin-2 content.

In a curved $3-d$ space-time there are two possible CS-type of terms; the rotational one L as given in eq. (1) and the translational (or triadic) CS-term T defined in the form

$$T = 2^{-1} m\kappa^{-2} < e_p{}^a \epsilon^{prs} \partial_r e_{sa} > . \tag{3}$$

T is neither locally conformal nor local Lorentz invariant. It is just diffeomorphism invariant. (Note that the bilinear type of CS-structure $\sim \omega\epsilon\partial e$ appears in Einstein's action).

Let us consider the possibility of spontaneously breaking the local Lorentz invariance of S eq. (1). This can be achieved by considering the diffeomorphism invariant action

$$S_{broken} = L - E + T \tag{4}$$

In order to analyse its content we take its quadratic part and go to Minkowski space-time $e_p{}^a = \delta_p{}^a + \kappa h_p{}^a$, $\omega_q{}^b = \kappa\omega_q{}^b$. S_{broken} becomes

$$\begin{aligned} S^0_{broken} = &(2\mu)^{-1} < \omega_p{}^a \epsilon^{prs} \partial_r \omega_{sa} > -2^{-1} < \omega_{pa}\omega^{ap} - \omega_a{}^a \omega_p{}^p > - \\ &-2^{-1} m < h_p{}^a \epsilon^{prs} \partial_r h_{sa} > + < \lambda_p{}^a \epsilon^{prs} (\partial_r h_{sa} - \omega_r{}^b \epsilon_{bsa}) >, \end{aligned} \tag{5}$$

where now $\lambda_p{}^a, \omega_p{}^a, h_p{}^a$ are independent variables.

Its equivalent third order version arises from introducing the values of $\omega = \omega(h)$ (obtained from variations of the λ's) into S_0.

Independent variations of ω, h, λ yield the triplet of field equations (FE).

$$E^p{}_a = \mu^{-1} \varepsilon^{prs} \partial_r \omega_{sa} - \omega_a{}^p + \delta_a{}^p \omega - \lambda_a{}^p + \delta_a{}^p \lambda = 0, \tag{6}$$

$$F^p{}_a = -m\varepsilon^{prs} \partial_r h_{sa} + \varepsilon^{prs} \partial_r \lambda_s{}^a = 0, \tag{7}$$

and

$$G^p{}_a = \varepsilon^{prs} \partial_r h_{sa} - \omega_a{}^p + \delta_a{}^p \omega = 0. \tag{8}$$

Considering the lower spin sector of these eqs., i.e. computing $E \equiv E^p{}_p$, F, G, $\partial_p E^p{}_a, \cdots$ and $\varepsilon_{pab} E^{pa} \equiv \check{E}_b$, $\check{F}_b$, $\check{C}_b$ it is straightforward to see that this system only propagates spin-2 excitations.

Both, the spin-1 $\varepsilon_{pab}\omega^{pa}, \cdots, \varepsilon_{pab}\lambda^{pa}, \partial_p\omega_{pa}, \partial_p\lambda_{pa}$ and the scalar sector of ω, h, λ vanish in the harmonic gauge $\partial_p h_{pa} = 0$.

Projection of the FE (7) (8) (9) upon the spin-2^+ (spin-2^-) subspaces using the pseudospin-$2^\pm$ projectors [7], gives

$$(X-1)\omega^{T+} - \lambda^{T+} = 0 \quad , \quad -mXh^{T+} + X\lambda^{T+} = 0 \quad , \quad \omega^{T+} = Xh^{T+} \tag{9}$$

where $X = \mu^{-1}\Box^{1/2}, \mu = 1$, m means the dimensionless relation $m\mu^{-1}$ and h^{T+} denotes the spin-2^+ part of h_{pa}.

The inverse propagator is therefore

$$\Delta^+(X) = X[X(X-1) - m]. \tag{10}$$

There is a positive mass $m = 2^{-1} + (4^{-1} + m)^{1/2}$ in the spin-2^+ sector. Similarly, since $\Delta^-(X) = X[X(X+1) - m]$ we might have a spin-2^- excitation with mass $m^- = -2^{-1} + (4^{-1} + m)^{1/2}$.

We want to see whether this system has its energy bounded from below (or not). It will be shown that, independently of the sign of m, the light-front (LF) generator is unbounded and consequently action (6) is physically meaningless, in spite of the fact that, from a covariant point of view, the system (7), (8), (9), seems to propagate two spin-2 decoupled excitations.

In order to have this, we calculate the value of the LF-generator of action (6) in terms of its two unconstrained variables ω_{vv} and λ_{vv}. Light front coordinates (u, v) are defined by

$$\eta^{11} = 1 = -\eta^{uv}, \quad u = 2^{-1/2}(x^0 - x^2), \quad v = 2^{-1/2}(x^0 + x^2), \quad \varepsilon^{1vu} = +1. \tag{11}$$

Time derivatives are written $\partial_u f = \dot{f}$ and the LF-spacelike ones are denoted $\partial_v f = f'$

One starts from the covariant expressions (6) of S_0 and express this action in terms of the 27 *LF*-field components $\omega_{uu} \equiv \omega_u$, ω_{uv}, ω_{vu}, $\omega_v \equiv \omega_{vv}$, $\omega_1 \equiv \omega_{11}$, ω_{1u}, ω_{u1}, ω_{1v} and $\omega_{v1}, \cdots, \lambda_u, \lambda_{uv}, \cdots, \lambda_{1v}, \lambda_{v1}$.

It is inmediate to realize that $\omega_{ua}, h_{ub}, \lambda_{uc}$ are multipliers associatted with nine differential constraint equations which can be solved, providing the values of $\omega_{1a}, h_{1b}, \lambda_{1c}$ as functions of the remaining nine intermediate variables $\omega_{va}, h_{vb}, \lambda_{vc}$. Their solution is:

$$\widehat{\omega}_{1v} = (\partial_1 + 1)\widehat{\omega}_v + \widehat{\lambda}_v \quad , \quad \widehat{h}_{1v} = \partial_1 \widehat{h}_v + \widehat{\omega}_v, \tag{12a, b}$$

$$\widehat{\lambda}_{1v} = \partial_1 \widehat{\lambda}_v + m\widehat{\omega}_v, \tag{12c}$$

$$\omega_1 = \partial_1 \widehat{\omega}_{v1} + \widehat{\omega}_{1v} + \widehat{\lambda}_{1v} \quad , \quad h_1 = \partial_1 \widehat{h}_{v1} + \widehat{\omega}_{1v}, \tag{13a, b}$$

$$\lambda_1 = \partial_1 \widehat{\lambda}_{v1} + m\widehat{\omega}_{1v}, \tag{13c}$$

$$\omega_{1u}' = (\partial_1 - 1)\omega_{vu} - \lambda_{vu} + \partial_1 \widehat{\omega}_{v1} + \partial_1 \widehat{\lambda}_{v1} + (m+1)\widehat{\omega}_{1v} + \widehat{\lambda}_{1v} \tag{14a}$$

$$h_{1u}' = \partial_1 h_{vu} - \omega_{vu} + \partial_1 \widehat{\omega}_{v1} + \widehat{\omega}_{1v} + \widehat{\lambda}_{1v} \tag{14b}$$

$$\lambda_{1u}' = \partial_1 \lambda_{vu} - m\omega_{vu} + m\partial_1 \widehat{\omega}_{v1} + m\widehat{\omega}_{1v} + m\widehat{\lambda}_{1v} \tag{14c}$$

where we introduced redefinitions like $\omega_{v1} \equiv \widehat{\omega}'_{v1}$, $\omega_v \equiv \widehat{\omega}''_v$, $\omega_{1v} \equiv \widehat{\omega}'_{1v}$ for the three sets of variables ω, λ, h.

In principle, the intermediate expression of S_0 obtained in terms of the nine intermediate variables $\omega_{va}, h_{vb}, \lambda_{vc}$ might have $\omega_{vu}, h_{vu}, \lambda_{vu}$ in the dynamical germ (the piece of $S_0 \sim p\dot{q}$).

However it turns out after using eqs. (13), (14) that these three variables are not present in this part of the action. While ω_{vu} and λ_{vu} constitute two additional Lagrange multipliers, h_{vu} has totally disappeared.

Independent variations of $\omega_{vu}, \lambda_{vu}$ lead to the final two differential constraints of S_0. Their solution shows the symmetry of the $1v$-components $\omega_{1v}, \lambda_{1v}$, i.e.

$$\widehat{\omega}_{v1} = \widehat{\omega}_{1v} \quad , \quad \widehat{\lambda}_{v1} = \widehat{\lambda}_{1v} \tag{15a, b}$$

Now it is immediate to obtain the unconstrained form of the evolution generator G of action $S_0 \sim p\dot{q} - G$. Since, at the initial stage when one writes down S_0 in terms of the LF-variables, G had the form:

$$G = < (\widehat{\omega}_{v1} + \widehat{\lambda}_{v1})\omega_{1v}' + \widehat{\omega}_{v1}\lambda_{1u}' >; \tag{16}$$

it is straightforward to realize that, after insertion of the values (15) of $\omega_{1u}', \lambda_{1u}'$ in it, G becomes:

$$G = < [(m+1)\widehat{\omega}_{1v} + \widehat{\lambda}_{1v}]^2 - m^2\widehat{\omega}_{1v}^2 > . \tag{17}$$

This explicitly shows that the generator is a non semidefinite positive quadratic expression. Consequently the unconstrained reduced form of S_0, even written in terms of the unique two gauge-invariant variables $\widehat{\lambda}_v, \widehat{\omega}_v$, does not have physical relevance. One can say that the presence of both types of CS terms is inconsistent. This situation is peculiar of Lorentz-CS gravity (there is no analogous third order CS theory for vector fields).

As we have a covariant first order action for studying this system, one can also perform a canonical newtonian $2+1$ type of analysis which leads to the analogous physical result; the energy is not definite positive. One might also wonder whether the pure double CS-gravitational action

$$S' = L + \varepsilon T \tag{18}$$

has some physical relevance. Its quadratic approximation on flat $3d$ Minkowski space consists of

$$\begin{aligned} S'^Q = (2\mu)^{-1} < {\omega_p}^a \epsilon^{prs} \partial_r \omega_{sa} > + \varepsilon 2^{-1} m < {h_p}^a \epsilon^{prs} \partial_r h_{sa} > + \\ + < {\lambda_p}^a \epsilon^{prs} (\partial_r h_{sa} - {\omega_r}^b \epsilon_{bsa}) > . \end{aligned} \tag{19}$$

Doing a similar analysis one finds out that the system only contains spin-2 excitations. However its energy is unbounded from below and consequently $S' = L + \varepsilon T$ has no physical relevance.

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