

# ON SUPERSYMMETRY AT HIGH TEMPERATURE

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## Abstract

While it is possible to find examples of field theories with a spontaneously broken symmetry at high temperature, in renormalizable supersymmetric models any internal symmetry gets always restored. Recently, a counterexample was suggested in the context of nonrenormalizable supersymmetric theories. We show that non negligible higher loop effects actually restore the symmetry, without compromising the validity of perturbation theory. We give some arguments as to why the proposed mechanism should not work in general.

## I. INTRODUCTION

In general, whether or not symmetries in field theory are broken at high temperature, is a dynamical question which depends on the parameter space of the theory in question. In spite of one's intuitive expectations on symmetry restoration [1], based on daily experience and proven correct in the simplest field theory systems [1–3], one can easily find examples with

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symmetries broken at high temperature [2,4]. This is an important issue, due to its possible role in the production of topological defects in the early universe. Symmetry nonrestoration at high temperature may provide a way out of both the domain wall [5,6] and the monopole problem [7–9].

A simple example of broken symmetry at high temperature is provided by supersymmetry. Due to the different boundary condition for bosons and fermions in thermal field theory, supersymmetry is automatically broken at any non-zero temperature. However, for the issue of topological defects, one would like to know what happens to internal symmetries in the context of supersymmetries. This question is nontrivial due to the highly constrained structure of SUSY models. It has been addressed carefully more than ten years ago [10]: in contrast with the non-supersymmetric case, it was shown that supersymmetry necessarily implies restoration of internal symmetries at high temperature. At least, this is what happens in renormalizable theories.

Recently, this conclusion was questioned by Dvali and Tamvakis [11] precisely by resorting to non-renormalizable potentials. They present an explicit example in which the inclusion of a quartic term in the superpotential allows apparently for non vanishing vevs at high temperature. Stimulated by their interesting suggestion, we have analyzed carefully their example, arriving however to the opposite conclusion. What happens, and what will be explained in detail below, is that the one-loop approximation used by them becomes invalid precisely due to the non-renormalizable nature of the superpotential. We find two-loop effects actually dominating the one-loop ones, and leading to symmetry restoration. We must stress that this is not due to a breakdown of perturbation theory, but rather a general feature of theories with more than one coupling, and in this sense it is completely analogous to the well known Coleman-Weinberg idea [12]. There, the one loop contribution to the Higgs self coupling due to gauge interactions may be as large as (if not larger than) the tree-level one. We believe this point will become clearer after the detailed discussion of our results.

The bulk of this paper is devoted precisely to this, in our opinion, important question. In the next section we give first a brief review of Dvali and Tamvakis work, and then present our findings. The idea raised in [11] can in fact be shown to provide a possible new and general mechanism, completely independent of SUSY, for symmetry nonrestoration at high temperature. We show in section III, on equally general grounds, why it cannot work, due to the necessarily dominating higher-loop effects.

## II. THE EXAMPLE: NON-RENORMALIZABLE WESS-ZUMINO MODEL

We take here the prototype model for symmetry nonrestoration of Ref. [11], which is basically a Wess-Zumino model with a discrete symmetry  $\mathbf{D} : \Phi \rightarrow -\Phi$  and the addition of a non-renormalizable interaction term:

$$W = -\frac{1}{2}\mu\Phi^2 + \frac{1}{4M}\Phi^4, \quad (1)$$

where  $M \gg \mu$ . This leads to the scalar potential

$$V = |\phi|^2 - \mu + \frac{\phi^2}{M}|^2$$

$$= \frac{\mu^2}{2}(\phi_1^2 + \phi_2^2) - \frac{\mu}{2M}(\phi_1^4 - \phi_2^4) + \frac{1}{8M^2}(\phi_1^2 + \phi_2^2)^3, \quad (2)$$

where  $\phi = (\phi_1 + i\phi_2)/\sqrt{2}$  is the scalar component of the chiral Wess-Zumino superfield  $\Phi$ . Notice that  $\phi_1$  has a negative quartic self coupling. At  $T = 0$ , as usual, one finds a set of two degenerate minima:  $\langle\phi\rangle = 0$  and  $\langle\phi\rangle^2 = \mu M$ . To see what happens at high  $T$ , in Ref. [11] the usual 1-loop induced correction to the effective potential is computed

$$\Delta V_{1\text{-loop}}(T) = \frac{T^2}{8} \left| \frac{\partial^2 W}{\partial \phi^2} \right|^2 = \frac{T^2}{8} \left| -\mu + \frac{3\phi^2}{M} \right|^2 \quad (3)$$

or

$$\Delta V_{1\text{-loop}}(T) = \frac{T^2}{8} \left[ \mu^2 - 3\frac{\mu}{M}(\phi_1^2 - \phi_2^2) + \frac{9}{4M^2}(\phi_1^2 + \phi_2^2)^2 \right]. \quad (4)$$

Dvali and Tamvakis conclude that for  $M^2 \gg T^2 \gg \mu M$ , one gets  $\langle\phi\rangle^2 \neq 0$ , as is immediately clear from (3).

Before we move on to question this statement, let us see what really is going on up to this point. As is transparent from (4), one can attribute (as usual) the symmetry breaking to the negative  $T^2$  mass term for  $\phi_1$ . However, the quartic self coupling of  $\phi_1$  in (2) being negative, one cannot ensure a nonvanishing vev. It is necessary to assume that either the  $\phi_1^6$  term in (2) or the  $\phi_1^4$  term in (4) (both positive, but suppressed by  $M^2$ ), dominates over the  $(\mu/M)\phi_1^4$  term, in order for  $\phi_1$  to have a vev.

Now, as we have required  $T^2 \gg \mu M$ , this is perfectly acceptable. But this is precisely where the problem lies: one has assumed that the non-renormalizable terms are not small in comparison with the renormalizable ones. Notice that this does not put in question the validity of perturbation theory, since the  $\phi^4$  terms are suppressed by the small parameter  $\mu/M$ . This is the analogy with the Coleman-Weinberg case that we drew before. Perturbation theory is perfectly safe, since the next term in the series would be of order  $\phi^8/M^4$ , or  $T^2\phi^6/M^4$ , which are strongly suppressed by  $T/M \ll 1$  or  $\phi/M \ll 1$ .

The question is: what about a term such as  $T^4\phi^2/M^2$ , which is obviously much bigger than  $(\mu/M)T^2\phi^2$ ? Notice that this is the only relevant term that one could have missed, and whose sign would decide the pattern of symmetry breaking, if present. Once again, the idea is to write the expansion in  $1/M$ , but due to the fact that one has two different couplings to start with (namely,  $(\mu/M)\phi^4$  and  $\phi^6/M^2$ ), one cannot resort to the usual loop expansion, since  $T^4\phi^2/M^2$  does not appear at one loop. Again, notice the complete parallel with the Coleman-Weinberg analysis. There, assuming a small self coupling  $\lambda$  for the Higgs field, one finds an important  $\phi^4$  term proportional to  $g^4$  ( $g$  being the gauge coupling constant) only at one loop, without implying the failure of perturbation theory.

We have found out that such a term,  $T^4\phi^2/M^2$ , does actually appear at the two loop level, through the diagrams depicted in Fig. 1.

Using the superpotential (1) and the usual rules for the evaluation of Feynman diagrams in thermal field theory [2,3], it is straightforward to calculate this contribution as

$$\Delta V_{2\text{-loops}}(T) = \frac{9T^4}{32M^2} |\phi|^2 = \frac{9T^4}{64M^2} (\phi_1^2 + \phi_2^2). \quad (5)$$

We wish to stress again that this term, in the range of parameters considered ( $M^2 \gg T^2 \gg \mu M$ ), dominates over the mass term in (4), and therefore must be taken into account. Since it is positive, the conclusion is contrary to the one in Ref. [11]: the discrete symmetry is restored at high temperature.

As we said before this result is valid up to leading order in an expansion in  $\phi/M$  and  $T/M$ . As long as we stay far away from  $M$ , the perturbation theory guarantees symmetry restoration. The reader may still feel uneasy about the consistency of calculations performed in a non-renormalizable theory. For this reason we have also performed the calculations in the renormalizable version of the theory. That is, as suggested in Ref. [11], one can consider a renormalizable superpotential

$$W = \frac{\mu}{2}\Phi^2 + \frac{M_X}{2}X^2 + \lambda X\Phi^2, \quad (6)$$

which upon integrating out the heavy field  $X$ , gives (1) after identifying  $M = M_X/2\lambda^2$ . Not surprisingly, considering all the graphs and in the limit  $M \gg T$ , the same correction (5) for the effective potential is obtained.

### III. GENERAL DISCUSSION

We have seen that, unfortunately, the suggestion put forward by the authors of Ref. [11] does not work. Here we should point out that the question raised by them has a much more general significance. Namely, if their idea were to work, this would pave a way to a new mechanism for avoiding symmetry restoration at high temperature, completely independently of supersymmetry.

To explain what is going on, let us recall the basics of symmetry nonrestoration at high temperature in renormalizable theories. Consider a theory with two scalar fields  $\phi_1, \phi_2$  and a potential symmetric under  $\mathbf{D}: \phi_1 \rightarrow -\phi_1, \phi_2 \rightarrow -\phi_2$

$$V(\phi_1, \phi_2) = \sum_{i=1}^2 \left( -\frac{m_i^2}{2}\phi_i^2 + \frac{\lambda_i}{4}\phi_i^4 \right) - \frac{\alpha}{2}\phi_1^2\phi_2^2 + \beta_1\phi_1^3\phi_2 + \beta_2\phi_2^3\phi_1. \quad (7)$$

The self couplings  $\lambda_i$  must be positive for the potential to be bounded from below, but one can always choose  $\alpha > 0, \beta_1, \beta_2 > 0$  in (7), and require  $\lambda_1\lambda_2 > \alpha^2$ . The high temperature corrections are

$$\Delta V_{1\text{-loop}}(T) = \frac{T^2}{24} \left[ (3\lambda_1 - \alpha)\phi_1^2 + (3\lambda_2 - \alpha)\phi_2^2 + 6(\beta_1 + \beta_2)\phi_1\phi_2 \right]. \quad (8)$$

By asking, e.g.  $\alpha > 3\lambda_1$ , one can keep one of the mass terms negative at any temperature, thus keeping the symmetry broken at any  $T$ . Notice that the signs of the scalar interactions and the corresponding  $T$ -dependent terms are equal. We have seen in section II that this feature persists in non-renormalizable theories.

In theories with a single field, this mechanism of nonrestoration would apparently be impossible, since the self-coupling must be positive in order to guarantee the boundedness of the potential. Here precisely enters the point of Ref. [11]: they have a negative quartic interaction for the  $\phi_1$  field but the theory is rendered stable through the positive non-renormalizable  $\phi_1^6$  term.

One could extend this mechanism to an arbitrary non-supersymmetric theory. To see why this cannot work in general, let us take the example of a real scalar field with a negative and small quartic self coupling, and a discrete symmetry  $\mathbf{D}:\phi \rightarrow -\phi$

$$V = \frac{\mu^2}{2}\phi^2 - \epsilon\phi^4 + \frac{\phi^{2n+4}}{M^{2n}}, \quad (9)$$

where we include the first important non-renormalizable term. The power  $n$  varies from model to model ( $n = 1$  in the case discussed above). The idea of Ref. [11] is based on two important points:  $\epsilon > 0$  and  $\epsilon \ll 1$ .

Notice that the non-renormalizable term makes the theory stable independently of the sign of  $\epsilon$ . At one loop level, one gets for  $T \ll M$  the corrections

$$\Delta V_{1\text{-loop}}(T) = \frac{T^2}{24} \left[ -12\epsilon\phi^2 + \frac{(2n+4)(2n+3)}{M^{2n}}\phi^{2n+2} \right]. \quad (10)$$

The idea is then that the temperature-induced non-renormalizable term is to combine with the one coming from the negative self-coupling to induce a vev when  $\Delta V(T)$  starts to dominate, i.e for  $T^2 \gg \mu^2$ . But of course, for this to happen one has to assume that the non-renormalizable term is not negligible, i.e.  $\epsilon$  very small. Here comes the point: if the non-renormalizable term is not negligible, one has to take into account its contributions to the thermal mass. This means that the expansion cannot end at one loop, but has to be pursued up to  $n+1$  loops. At that level, the ‘‘butterfly’’ diagrams with  $n+1$  loops and two external legs of which Fig. 1 (a) is the  $n = 1$  example, will induce the high temperature contribution

$$\Delta V_{n+1\text{-loops}}|_{\text{mass term}}(T) = \frac{1}{2} \left( \frac{T^2}{12} \right)^{n+1} \frac{(2n+4)!}{2^{n+1}(n+1)!} \frac{1}{M^{2n}} \phi^2. \quad (11)$$

Any other term in the expansion of the couplings  $1/M^{2n}$  and  $\epsilon$  will be suppressed. Each loop in the diagram will provide a positive contribution  $T^2/12$ , so the sign of (11) is the sign of the coupling. A positive mass term already indicates that the symmetry will be restored, however one should look at all the temperature-dependent interactions that follow from the non-renormalizable terms. The diagrams that give the dominant  $1/M^{2n}$  contribution to the  $\phi^{2m}$  interaction terms are again the ‘‘butterflies’’ with  $2m$  external legs, and they are readily calculated

$$\Delta V(T) = \sum_{m=1}^{n+1} \frac{(2n+4)!}{(2m)!(n-m+2)!2^{n-m+2}} \left( \frac{T^2}{12} \right)^{n-m+2} \frac{\phi^{2m}}{M^{2n}}. \quad (12)$$

All the terms of the series have a positive sign, not surprisingly, as we mentioned before the high-T contributions carry the sign of the coupling constant. Symmetry restoration then follows.

We can easily generalize (12) to get the ‘‘butterfly’’ contribution to the high temperature effective potential of an arbitrary  $V(\phi)$ :

$$V(\phi, T) = \sum_{m=0}^{\infty} \frac{1}{m!} \left( \frac{T^2}{24} \right)^m \left\{ \frac{d^{2m}V}{d\phi^{2m}}(\phi) - \frac{d^{2m}V}{d\phi^{2m}}(\phi=0) \right\}. \quad (13)$$

## IV. CONCLUSIONS

According to our results, the idea of Ref. [11] of using higher dimensional effective interactions to provide nonrestoration of internal symmetries in a supersymmetric context seems not to work. This, in turn, would confirm the general result [10] which was proved only for renormalizable supersymmetric theories.

We have also offered arguments of why we believe this to hold in general. However, admittedly, we do not have a rigorous proof of symmetry restoration in the multifield non-renormalizable supersymmetric case. The purpose of our paper is in a sense twofold: first, since the issue is so important, it was crucial to know whether the discussed mechanism for nonrestoration [11] is valid or not; and second, we hope that it may inspire the reader to provide either a rigorous proof of the no-go theorem [10], or a way out.

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## REFERENCES

- [1] D.A. Kirzhnits, *JETP Lett.* **15** (1972), 529; D.A.Kirzhnits and A.D. Linde, *Phys. Lett.* **B42** (1972) 47.
- [2] S.Weinberg, *Phys. Rev.*, **D9**, (1974) 3357.
- [3] L. Dolan and R. Jackiw, *Phys. Rev.*, **D9**, (1974) 3320.
- [4] R.N. Mohapatra and G.Senjanović, *Phys. Rev.Lett*, **42**, (1979) 1651; *Phys. Rev.* **D20**, (1979) 3390.
- [5] G. Dvali and G. Senjanović, *Phys. Rev. Lett.* **74** (1995) 5178.
- [6] G. Dvali, A. Melfo and G. Senjanović “Nonrestoration of spontaneously broken P, CP and PQ at high temperature”, SISSA Ref. 18/96/A, hep-ph 9601376.
- [7] P.Langacker and S.-Y. Pi, *Phys. Rev. Lett.* **45**, (1980)
- [8] P. Salomonson, B.-S. Skagerstam and A. Stern, *Phys. Lett.* **B151** (1985) 243.
- [9] G. Dvali, A. Melfo and G. Senjanović, *Phys. Rev. Lett.* **75**, 4559 (1995).
- [10] H. Haber, *Phys. Rev.* **D 46** (1982) 1317 ; M. Mangano *Phys.Lett.* **147B** (1984) 307
- [11] G. Dvali and K. Tamvakis, “ Symmetry Non-restoration at High Temperature and Supersymmetry”, CERN-TH/96-45, hep-th 9602336.
- [12] S. Coleman and E. Weinberg, *Phys. Rev.* **D 7**, 1888, 1973.

FIGURES

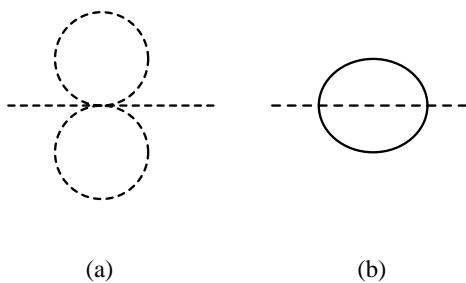


FIG. 1. Feynman diagrams for the thermal mass correction from the non-renormalizable term. Dashed lines represent the scalar boson  $\phi$ , continuous lines represent its fermion counterpart  $\tilde{\phi}$ .