

MASSIVE GAUGE AXION FIELDS

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Abstract

A gauge invariant formulation for the massive axion is considered. The axion acquires mass through a topological term which couples a (pseudo)scalar and a third rank antisymmetric tensor. Duality, local and canonical equivalences with the non-gauge invariant proposal are established. The supersymmetric version of the gauge invariant model is constructed.

It is well known that different theories can be related by duality transformations and their classical and quantum properties are equivalent. In four dimensions a massless (pseudo)scalar field: the axion, is dual to the antisymmetric field B_{mn} (only if derivative couplings are considered). This fact is a particular case of the general duality between p and $D - p - 2$ forms in D dimensions. In this sense, it is known that non-perturbative effects break the Pecci-Quinn (PQ) symmetry of the axion, giving mass to it. Then, a natural question arises, concerning about what happens with the duality symmetry. This item was an enigma until two independent approaches were developed recently [1], [2]. In these references the authors show that, though the PQ symmetry is broken, duality symmetry still remains. The B_{mn} field is replaced by a massive antisymmetric field of third rank C_{mnp} . A characteristic feature of this duality is the lost of abelian gauge invariance for the new antisymmetric field. The duality now corresponds to a particular case of the usual duality between massive p and $D - p - 1$ forms in D dimensions.

The connection between a massive (pseudo)scalar field, ϕ , and a massive antisymmetric field, C_{mnp} has been considered before in different contexts

[3][4][5][6]. In this letter, we will show that a gauge invariant model, which involves a topological coupling, considered in reference [4], in the context of the $U(1)$ problem, is locally equivalent to the non gauge invariant proposal for the massive axion. This equivalence is analogous to that between the “self-dual” [7] and topologically massive [8] models in three dimensions, in the same manner as the equivalence between the Proca and the topologically massive gauge invariant theories in four dimensions [9] [10]. In the preceding examples, in three and four dimensions, the connection occurs between a gauge sensitive and a gauge invariant model. It has been shown that these models are locally equivalent and one of them is a gauge fixed version of the other [11][9]. Nevertheless the global equivalence depends on the cohomological structure of the base manifold [12][9][13][14]. For the models that we compare in this letter we will face this dual equivalence in several ways. One of them, described below by I_{M1} , is a first order version of the massive (pseudo)scalar model. The other model, described below by I_{M2} , is gauge invariant. We will note that the space of solutions of both models differ in a topological sector that can be gauged out on trivial manifolds. The presence of this topological sector appears explicitly in the partiton function of the gauge invariant model as a topological factor related with the cohomological structure of the base manifold. We will prove (canonically) that one of the models is a gauge fixed version of the gauge invariant one. In particular, we will see the equivalence through the existence of a first order master action, that will allow us to consider global aspects and interactions. Finally we will construct a supersymmetric formulation of the gauge invariant action of the massive axion.

An illustrative model for the massive axion is given by the following master action[5] [15]

$$I = \langle \frac{1}{2}v_m v^m + \phi \partial_m v^m - \frac{1}{2}m^2 \phi^2 \rangle, \quad (1)$$

where v_m is a vector field and ϕ is a scalar field ($\langle \rangle$ denotes integration in four dimensions). Eliminating the field v_m through its equation of motion ($v_m = \partial_m \phi$), the action for a massive scalar is obtained, while using the equation of motion of the scalar field ($\phi = \frac{1}{m^2} \partial_m v^m$), we have

$$I_v = \frac{1}{2} \langle v_m v^m + \frac{1}{m^2} (\partial_m v^m)^2 \rangle. \quad (2)$$

The propagator corresponding to the field v_m is $\eta_{mn} - \frac{k_m k_n}{k^2 + m^2}$, which is just equal to those discussed in [1]. If we introduce the dual of the vector field: $v^m = \frac{1}{3!} m \epsilon^{mnpq} C_{npq}$, we have the following master action

$$I_{M1} = \langle -\frac{1}{2 \cdot 3!} m^2 C^{mnp} C_{mnp} + \frac{1}{3!} m \epsilon^{mnpq} \phi \partial_m C_{npq} - \frac{1}{2} m^2 \phi^2 \rangle, \quad (3)$$

from which duality is easily inferred. To see this, we can use its equations of motion

$$m C^{mnp} - \epsilon^{mnpq} \partial_q \phi = 0, \quad (4)$$

$$m \phi - \frac{1}{3!} \epsilon^{mnpq} \partial_m C_{npq} = 0, \quad (5)$$

to eliminate C_{mnp} (or ϕ), thus obtaining the action for a massive scalar field (or the massive antisymmetric field C_{mnp}). In any case, there is not gauge invariance at all. We can ask whether there really exist a gauge invariant theory compatible with a massive term for the axion field. The answer is positive, and the action is

$$I_{M2} = \langle -\frac{1}{2} \partial_m \phi \partial^m \phi - \frac{1}{2 \cdot 4!} G_{mnpq} G^{mnpq} - \frac{m}{3!} \epsilon^{mnpq} C_{mnp} \partial_q \phi \rangle, \quad (6)$$

where $G_{mnpq} \equiv \partial_m C_{npq} - \partial_n C_{mpq} + \partial_p C_{mnq} - \partial_q C_{mnp}$ is the field strength associated with the antisymmetric field C_{mnp} . This action was considered previously in reference [4] as a generalization to four dimensions of the Schwinger model in two dimensions. This action is invariant, up to a surface term, under the abelian gauge transformations

$$\delta_\xi C_{mnp} = \partial_m \xi_{np} + \partial_n \xi_{pm} + \partial_p \xi_{mn}, \quad \delta_\xi \phi = 0. \quad (7)$$

and under the global transformation: $\delta \phi = \text{constant}$. In I_{M2} we note that the coupling term is a BF topological term. This term preserves the number of degrees of freedom of the non-interacting terms (just the one connected with the massless (pseudo)scalar field), but change the physical characteristic of the spectrum (the (pseudo)scalar field becomes massive). This situation is like the one in three dimension with the topologically massive model where the topological Chern-Simons term changes the physical properties of the Maxwell field (which in three dimensions is a massless scalar) to a massive spin 1 theory. Other example of this situation, in four dimensions, occurs in

the Cremmer-Scherk[16] or topologically massive model where the original fields are a massless scalar (one degree of freedom) and a photon with helicity 1 (two degrees of freedom) and the resulting theory with the topological interaction term is a massive spin 1 excitation (three degrees of freedom).

The connection of I_{M_2} and the massive (pseudo)scalar is straightforward: introducing $\lambda \equiv -\frac{1}{4}\epsilon^{mnpq}G_{mnpq}$ as the dual of the strength field G_{mnpq} into the action I_{M_2} , we observe that λ plays the role of an auxiliary field, whose elimination through its equation of motion ($\lambda = -m\phi$) leads to the action of a massive (pseudo)scalar field.

It is worth, at this point, to note that since I_{M_2} depends only of derivatives of the scalar field, then its dual theory can be achieved, replacing $\partial_m\phi$ by $\frac{1}{2}l_m$ and adding a BF term: $\frac{1}{4}l_m\epsilon^{mnpq}\partial_n B_{pq}$ [17]. The dual action will be [4][6]

$$I_d = \langle -\frac{1}{2.4!}G_{mnpq}G^{mnpq} - \frac{1}{2.3!}(mC_{mnp} - H_{mnp})(mC^{mnp} - H^{mnp}) \rangle, \quad (8)$$

where $H_{mnp} = \partial_m B_{np} + \partial_n B_{pm} + \partial_p B_{mn}$ is the field strength of the antisymmetric field B_{mn} , which was introduced in the BF term. This action describes the interaction of open membranes whose boundaries are closed strings [18] and is invariant under the following gauge transformations

$$\delta C_{mnp} = \partial_m \xi_{np} + \partial_n \xi_{pm} + \partial_p \xi_{mn}, \quad \delta B_{mn} = \partial_m \lambda_n - \partial_n \lambda_m - m\xi_{mn}. \quad (9)$$

The ξ gauge transformation allows us gauged away the antisymmetric field B_{mn} , leading to the massive antisymmetric field C_{mnp} action.

Let us, now, see that I_{M_2} is locally equivalent to I_{M_1} on-shell. The equations of motion for I_{M_2} can be written, suggestively, as

$$\epsilon^{mnpq}\partial_q(mC_{mnp} - \epsilon_{mnp r}\partial^r\phi) = 0, \quad (10)$$

$$\epsilon^{mnpq}\partial_q(m\phi - \frac{1}{3!}\epsilon^{rstu}\partial_r C_{stu}) = 0. \quad (11)$$

These equations, are just the result of applying the differential operator $\epsilon^{mnpq}\partial_q$ to the equations of motion derived from I_{M_1} : eqs. (4) and (5). From here the local equivalence can be stated: equations (10) and (11) can be written locally as

$$mC_{mnp} - \epsilon_{mnp r}\partial^r\phi = \partial_m\lambda_{np} + \partial_n\lambda_{pm} + \partial_p\lambda_{mn}, \quad (12)$$

and

$$m\phi - \frac{1}{3!}\epsilon^{mnpq}\partial_m C_{npq} = constant. \quad (13)$$

Now, the right sides of (12) and (13) correspond, respectively, to gauge changes in C_{mnp} and ϕ . So those sides are gauge equivalent to zero, leaving us with equations (4) and (5). In other words the physical relevant solutions of both models are the same, at the local level. Equivalently, closed 0 and 3-forms ($\partial_m \phi = 0 = \partial_{[m} C_{npq]}$) are always solutions of the equations (10) and (11), while they become trivial in the model described by I_{M1} . Locally, this closed forms are always exact and then the gauge invariance allow us to gauge them away. Globally, this will depend on the cohomological structure of the base manifolds and both models may differ in their space of solutions. We will see later that this topological dependence will appear explicitly in the partition function.

Proceeding with the comparison we will show the canonical equivalence for these two theories on cohomological trivial region of space-time. For the non-gauge invariant action I_{M1} , the canonical hamiltonian density is found to be

$$\mathcal{H}_1 = \frac{1}{2 \cdot 3!} m^2 C_{ijk} C_{ijk} + \frac{1}{2} m^2 \phi^2 + \frac{1}{2} \partial_i \phi \partial_i \phi, \quad (14)$$

subject to the set of second class constraints

$$\psi = 0 \quad \text{and} \quad \psi_{ijk} = \pi_{ijk} - \frac{1}{3!} m \epsilon_{ijk} \phi, \quad (15)$$

where π and π_{ijk} are the canonical conjugated momenta associated to ϕ and C_{ijk} , respectively.

On the other hand, for the gauge invariant action I_{M2} , we have

$$\mathcal{H}_2 = 3\pi_{ijk}\pi_{ijk} + \frac{1}{2} \partial_i \phi \partial_i \phi + \frac{1}{2} [\pi - \frac{m}{3!} \epsilon^{ijk} C_{ijk}]^2, \quad (16)$$

subject to the first class constraints

$$\theta_{jk} = \partial_i \pi_{ijk} - \frac{1}{3!} m \epsilon^{ijk} \partial_i \phi. \quad (17)$$

These constraints are doubly reducible, i.e., there are functions $Z_k^{ij}(x, y)$ and $Z_k(x, y)$ such that $\int dy Z_k^{ij}(x, y) \theta_{ij}(y) = 0$ and $\int dy Z^k(x, y) Z_k^{ij}(x, z) = 0$. This fact must be taken into account when we want the gauge freedom to be fixed.

It is straightforward to show that these two hamiltonian densities are related by linear combinations of the set of second class constraints of \mathcal{H}_1 ,

reflecting the canonical equivalence:

$$\mathcal{H}_2 = \mathcal{H}_1 + 3\psi_{ijk}[\psi_{ijk} + \frac{1}{3}m\epsilon_{ijk}\phi] + \frac{1}{2}\psi[\psi - \frac{1}{3}m\epsilon_{ijk}C_{ijk}]. \quad (18)$$

This situation is similar to those theories which involve a mass mechanism through topological coupling [11][9][12]. Moreover, we can see that I_{M1} is a gauge fixed version of the gauge invariant action I_{M2} . In fact, we note that $\theta_{ij} = \partial_k\psi_{ijk}$ can be considered as the first class constraints and $\psi = 0$ as the gauge fixing condition¹. In the latter case the first order hamiltonian results to be H_2 . The canonical procedure continues at this point and then the quantum equivalence is clear.

Now, in order to touch further aspects it is worth looking at the equivalence through the following master action

$$\begin{aligned} I_M = & < -\frac{1}{2 \cdot 3!}m^2 a_{mnp}a^{mnp} - \frac{1}{2!}m^2\psi^2 + \frac{1}{4!}m\epsilon^{mnpq}\psi G_{mnpq} \\ & + \frac{1}{3!}m\epsilon^{mnpq}(a_{mnp} - C_{mnp})\partial_q\phi > . \end{aligned} \quad (19)$$

Independent variations in a_{mnp} , ψ , C_{mnp} and ϕ lead to the following equations of motion

$$a^{mnp} = \frac{1}{m}\epsilon^{mnpq}\partial_q\phi, \quad (20)$$

$$\psi = \frac{1}{4!m}\epsilon^{mnpq}G_{mnpq}, \quad (21)$$

$$\epsilon^{mnpq}\partial_m(\psi - \phi) = 0 \quad (22)$$

and

$$\epsilon^{mnpq}\partial_q(a_{mnp} - C_{mnp}) = 0. \quad (23)$$

Replacing the expressions for a_{mnp} and ψ given by eqs. (20) and (21) into I_M , the gauge invariant action I_{M2} is obtained. On the other hand, the solutions of the equations of motion (22) and (23) are

$$\phi - \psi = \omega, \quad C_{mnp} - a_{mnp} = \Omega_{mnp}, \quad (24)$$

where ω and Ω_{mnp} are 0 and 3-closed forms, respectively. Locally, we can set

$$\omega = \text{constant}, \quad \Omega_{mnp} \equiv L_{mnp} = \partial_m l_{np} + \partial_n l_{pm} + \partial_p l_{mn}, \quad (25)$$

¹ ψ_{ijk} and $\partial_i\psi_{ijk}$ have the same information in three space dimensions

and substituting into I_M , we obtain the following ‘‘Stüeckelberg’’ type action

$$I_s = < -\frac{1}{2 \cdot 3!} m^2 (C_{mnp} - L_{mnp})(C^{mnp} - L^{mnp}) - \frac{1}{2} m^2 (\phi - \omega)^2 + \frac{1}{4!} m \epsilon^{mnpq} (\phi - \omega) G_{mnpq} > . \quad (26)$$

This action is invariant under

$$\delta_\xi C_{mnp} = \partial_m \xi_{np} + \partial_n \xi_{pm} + \partial_p \xi_{mn}, \quad \delta_\xi l_{mn} = \xi_{mn}, \quad (27)$$

which allow us to gauge away the l_{mn} field and recover I_{M1} (we have redefined $\phi - \omega$ as ϕ since ω is a constant). In this way, the local equivalence is also stated. On the other hand, we can consider $\psi = \phi - \omega$ and $a_{mnp} = C_{mnp} - \Omega_{mnp}$ as general solutions in order to obtain the following gauge invariant action

$$\bar{I}_M = < -\frac{1}{2 \cdot 3!} m^2 (C_{mnp} - \Omega_{mnp})(C^{mnp} - \Omega^{mnp}) - \frac{1}{2!} m^2 (\phi - \omega)^2 + \frac{1}{4!} m \epsilon^{mnpq} (\phi - \omega) G_{mnpq} - \frac{1}{3!} m \epsilon^{mnpq} \Omega_{mnp} \partial_q \phi > . \quad (28)$$

This action is global and locally equivalent to I_{M2} , so the topological sectors, not present in I_{M1} , are now included. Indeed, the equations of motion which are obtained after performing independent variations on $C_{mnp}, \Omega_{mnp}, \phi$ and ω in \bar{I}_M are

$$\begin{aligned} m(C^{mnp} - \Omega^{mnp}) - \epsilon^{mnpq} \partial_q (\phi - \omega) &= 0 \\ m(C^{mnp} - \Omega^{mnp}) - \epsilon^{mnpq} \partial_q \phi &= 0 \end{aligned} \quad (29)$$

and

$$\begin{aligned} m(\phi - \omega) - \frac{1}{3!} \epsilon^{mnpq} \partial_m C_{npq} + \frac{1}{3!} \epsilon^{mnpq} \partial_m \Omega_{npq} &= 0 \\ m(\phi - \omega) - \frac{1}{3!} \epsilon^{mnpq} \partial_m C_{npq} &= 0, \end{aligned} \quad (30)$$

from which is easily deduced that ω and Ω_{mnp} are closed 0 and 3-forms, respectively, i.e. $\epsilon^{mnpq} \partial_q \omega = 0 = \epsilon^{mnpq} \partial_m \Omega_{npq}$. Taking into account this last result and applying the differential operator $\epsilon^{mnpq} \partial_q$ on the set first order differential equations given by (30), the equations of motion for the gauge

invariant action I_{M2} are obtained (equations (10) and (11)). We can say that \bar{I}_M is the correct modification to I_{M1} in order to include the, originally missing, topological sectors. This aspect will be important when interactions are to be considered. Moreover, we can eliminate ϕ and C_{mnp} to achieve

$$\bar{I}_{M1} = I_{M1[a,\psi]} - I_{top[\omega,\Omega]}, \quad (31)$$

where

$$I_{top[\omega,\Omega]} = \langle \frac{1}{3!} m \epsilon^{mnpq} \Omega_{mnp} \partial_q \omega \rangle \quad (32)$$

is the BF term for the topological coupling between 0 and 3-forms in four dimensions. In other direction, if we couple a source to ϕ and C_{mnp} only, in (28), it is easy to see that the sum of any solution of the inhomogeneous equations of I_{M1} with any solution of the inhomogeneous equation of the BF theory is a solution of the inhomogeneous equation of I_{M2} .

From (32) we have that the partition functions of I_{M1} and I_{M2} differ by a topological factor.

$$Z_{M2} = Z_{top} Z_{M1} \quad (33)$$

This topological factor is associated with the topological sectors not present in the space of solutions of I_{M1} . In general, on manifolds with non trivial topological structure $Z_{top} \neq 1$. Only when the manifold has a trivial structure, we will have $Z_{top} \equiv 1$, reflecting the local and global equivalence (In the cases of duality equivalence taking into account global aspects see references [19] [20]).

Finally, we will construct the supersymmetric formulation of the gauge invariant action for the massive axion. For this proposal, we need (anti)chiral superfields (Φ and Φ^+) and a superfield V whose supersymmetric field strength is $G = -\frac{1}{2}(DD + \bar{D}\bar{D})V$, where (DDV) $\bar{D}\bar{D}$ is (anti) chiral superfield. This later superfield describes the C_{mnp} in a supersymmetric way[21]. The supersymmetric action is

$$\begin{aligned} I &= \frac{1}{16} \int d^4x d^2\theta d^2\bar{\theta} G^2 + \frac{1}{2} \int d^4x d^2\theta d^2\bar{\theta} \Phi \Phi^+ \\ &+ \frac{1}{4} m \int d^4x d^2\theta \Phi \bar{D}\bar{D}V + \frac{1}{4} m \int d^4x d^2\bar{\theta} \Phi^+ DDV. \end{aligned} \quad (34)$$

where the last two terms are the supersymmetric extension of the topological coupling. In components (using Majorana spinors), this action is written

down as

$$\begin{aligned}
I &= -\frac{1}{2}\partial_m A\partial_m A - \frac{1}{2}\partial_m B\partial_m B + \frac{1}{2}(\partial_m v_m)^2 + \frac{1}{2}d^2 - \frac{i}{2}\bar{\lambda}\gamma^m\partial_m\lambda \\
&- \frac{1}{2}\partial_m a\partial_m a - \frac{1}{2}\partial_m\phi\partial_m\phi - \frac{i}{2}\bar{\chi}\gamma^m\partial_m\chi + \frac{1}{2}f^2 + \frac{1}{2}g^2 \\
&- m[\phi\partial_m v_m + ad + i\bar{\chi}\gamma_5\lambda - (fB - gA)],
\end{aligned} \tag{35}$$

where $(A, B, v^m \equiv \frac{1}{3!}\epsilon^{mnpq}C_{npq}, d, \lambda)$ and (a, ϕ, χ, f, g) are the components of V and Φ multiplets, respectively. This action describes 8 bosonic and 8 fermionic degrees of freedom and is invariant under the off-shell supersymmetric transformations

$$\begin{aligned}
\delta A &= i\bar{\epsilon}\lambda, & \delta B &= i\bar{\epsilon}\gamma_5\lambda, \\
\delta v &= i\bar{\epsilon}\gamma^m\lambda, & \delta d &= i\bar{\epsilon}\gamma_5\gamma^m\partial_m\lambda, \\
\delta\lambda &= \epsilon(\partial\cdot v) + \gamma_5\epsilon d + \gamma^m\epsilon\partial_m A - \gamma_5\gamma^m\epsilon\partial_m A
\end{aligned} \tag{36}$$

and

$$\begin{aligned}
\delta a &= i\bar{\epsilon}\chi, & \delta\phi &= i\bar{\epsilon}\gamma_5\chi, \\
\delta\chi &= [\partial_m a - \gamma_5\partial_m\phi]\gamma^m\epsilon + [f + \gamma_5g]\epsilon, \\
\delta f &= i\bar{\epsilon}\gamma^m\partial_m\chi, & \delta g &= i\bar{\epsilon}\gamma_5\gamma^m\partial_m\chi.
\end{aligned} \tag{37}$$

Note that the a, A and B bosonic fields also have the same mass as the axion field, as can be seen after eliminating the g, d and f auxiliary fields, just as the dilaton acquire mass in the linear multiplet, in the supersymmetric formulation for the Cremmer-Sherk action[22]

Sumarizing, we have seen that a gauge invariant description for massive axions is possible. This gauge invariant description is (locally)equivalent to the non-gauge invariant proposal. The supersymmetric formulation was given. A complete BRST analysis of the gauge invariant model considered in this paper is under consideration.

ACKNOWLEDGEMENTS

One of the authors (AK) would like to thank to the Consejo de Desarrollo Científico y Humanístico de la Universidad de los Andes(CDCHT-ULA) by institutional support under project C-862-97.

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